Lesson 566 – Recommended review and AP Practice

63. ***Atmospheric Pressure*** Atmospheric pressure P (measured in millimeters of mercury) decreases exponentially with increasing altitude x (measured in meters). The pressure is 760 millimeters of mercury at sea level ($x = 0$) and 672.71 millimeters of mercury at an altitude of 1000 meters. Find the pressure at an altitude of 3000 meters.

65. ***Learning Curve*** The management at a certain factory has found that a worker can produce at most 30 units in a day. The learning curve for the number of units N produced per day after a new employee has worked t days is

$N = 30(1 - e^{kt})$.

After 20 days on the job, a particular worker produces 19 units.

(a) Find the learning curve for this worker.

(b) How many days should pass before this worker is producing 25 units per day?

66. ***Learning Curve*** If in Exercise 65 management requires a new employee to produce at least 20 units per day after 30 days on the job, find (a) the learning curve that describes this minimum requirement and (b) the number of days before a minimal achiever is producing 25 units per day.

20. The average value of $f(x)=\frac{1}{x}$ from $x=1$ to $x=e$ is

4.4

(A) $\frac{1}{e+1}$

(B) $\frac{1}{1-e}$

(C) $e-1$

(D) $1-\frac{1}{e^2}$

(E) $\frac{1}{e-1}$

5.4

C

80. If the line tangent to the graph of $f(x)=e^{3x^2}$ at the point $(a,\ f(a))$ is parallel to the line $y=3x-5$, then $a=$

(A) -0.605

(B) -0.183

(C) 0.183

(D) 0.348

(E) 0.605

5.4

C

44. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t)=7e^{-t^2}$; where t stands for time.

(A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

16. $\int 7xe^{3x^2}dx=$ 5.4

(A) $\frac{1}{42}e^{3x^2}+C$

(B) $\frac{6}{7}e^{3x^2}+C$

(C) $\frac{7}{6}e^{3x^2}+C$

(D) $7e^{3x^2}+C$

(E) $42e^{3x^2}+C$

C

30. The average value of $f(x)=e^{4x^2}$ on the interval $\left[-\frac{1}{4}, \frac{1}{4}\right]$ is 4.4

(A) 0.272

(B) 0.545

(C) 1.090

(D) 2.180

(E) 4.360

* hard

5. Let f be the function defined by $f(x) = e^x(ax^2 + bx + 6)$, where a and b are constants. The function f has a relative extremum at the point $x = -1$, and $f(-1) = 17e^{-1}$.

(a) Find a and b. 5.4

(b) Is the point $(-1, 17e^{-1})$ a relative minimum or a relative maximum of f? 5.4

(c) Find all the points of inflection of the graph of f. 5.4

19. $\int \left(e^{3\ln x} + e^{3x}\right) dx =$ 5.4

(A) $3+\frac{e^{3x}}{3}+C$

(B) $\frac{x^4}{4}+3e^{3x}+C$

(C) $\frac{e^{x^4}}{4}+3e^{3x}+C$

(D) $\frac{e^{x^4}}{4}+\frac{e^{3x}}{3}+C$

(E) $\frac{x^4}{4}+\frac{e^{3x}}{3}+C$

C

84. What is the x-value of the point at which the tangent line to the graph of $y = e^{x^2} + x$ perpendicular to the line $3y = 2x + 1$? 5.4

(A) −0.732

(B) −0.589

(C) −0.162

(D) 0.236

(E) 0.361

C 4.4

31. $\int_0^1 \tan x \, dx =$

(A) 0

(B) $\frac{\tan^2 1}{2}$

(C) $\ln(\cos(1))$

(D) $\ln(\sec(1))$

(E) $\ln(\sec(1)) - 1$

6. An object moves with velocity $v(t) = t^2 - 8t + 7$. 4.4

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$. 4.1

(b) At what time(s) is the particle changing direction? 3.3

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 4$. 4.4

C 5.6

44. A radioactive isotope, y, decays according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in seconds. If the half-life of y is 1 minute, then the value of k is

(A) −41.589 (B) −0.012 (C) 0.027 (D) 0.693 (E) 98.923

C

92. A population of bacteria given by $y(t)$ grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in minutes. If $y(10) = 10$ and $y(30) = 25$, what is the value of k? 5.6

(A) −2.079

(B) 0.046

(C) 0.107

(D) 0.125

(E) 0.230

C

3. Sea grass grows on a lake. The rate of growth of the grass is $\frac{dG}{dt} = kG$, where k is a constant.

   (a) Find an expression for G, the amount of grass in the lake (in tons), in terms of t, the number of years, if the amount of grass is 100 tons initially, and 120 tons after one year.

   (b) In how many years will the amount of grass available be 300 tons?

   (c) If fish are now introduced into the lake and consume a consistent 80 tons/year of sea grass, how long will it take for the lake to be completely free of sea grass?

now is at
the end
of year 1

C

2. At time $t = 0$ minutes, the temperature of a cup of coffee is 180 degrees Fahrenheit. Left in a room whose temperature is 70 degrees Fahrenheit, the coffee cools so that its temperature function $T(t)$, also measured in degrees Fahrenheit, satisfies the differential equation $\frac{dT}{dt} = -\frac{1}{2}T + 35$.

   (a) Find an expression for $T(t)$ using the initial condition $T(0) = 180$.

   (b) Find $\lim_{t\to\infty} T(t)$. Explain what this limit means in the context of the problem.

   (c) At what time t is the temperature of the coffee decreasing at the rate of 15 degrees Fahrenheit per minute? How hot is the coffee at that point? Indicate units of measurement.

20. The average value of $f(x)=\frac{1}{x}$ from $x=1$ to $x=e$ is

$\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx = \frac{1}{e-1}[\ln e - \ln 1]$

$=\frac{1}{e-1}[1-0]$

$=\frac{1}{e-1}$

4.4

(A) $\frac{1}{e+1}$

(B) $\frac{1}{1-e}$

(C) $e-1$

(D) $1-\frac{1}{e^2}$

(E) $\frac{1}{e-1}$

~~22~~

5.4

C

80. If the line tangent to the graph of $f(x)=e^{3x^2}$ at the point $(a,\ f(a))$ is parallel to the line $y=3x-5$, then $a=$

$\bar{\bar{m}}$

$f'(a)=3$

(A) -0.605

(B) -0.183

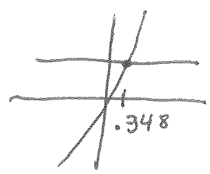
(C) 0.183

(D) 0.348

(E) 0.605

$f'(x)=e^{3x^2}\cdot(6x)$

$3=e^{3x^2}\cdot 6x$



5.4

C

44. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t)=7e^{-t^2}$; where t stands for time.

(A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

$\int_0^4 7e^{-t^2}\,dt$

5.4

16. $\int 7xe^{3x^2}\,dx=$

(A) $\frac{1}{42}e^{3x^2}+C$

(B) $\frac{6}{7}e^{3x^2}+C$

(C) $\frac{7}{6}e^{3x^2}+C$

(D) $7e^{3x^2}+C$

(E) $42e^{3x^2}+C$

$u=3x^2$

$du=6x\,dx$

$\frac{7}{6}du=7x\,dx$

$\int e^u\cdot\frac{7}{6}du$

$\frac{7}{6}\cdot\int e^u du=\frac{7}{6}e^u+C$

$=\frac{7}{6}e^{3x^2}+C$

C 4, 4

30. The average value of $f(x)=e^{4x^2}$ on the interval $\left[-\frac{1}{4},\frac{1}{4}\right]$ is

(A) 0.272
(B) 0.545
(C) 1.090
(D) 2.180
(E) 4.360

$\frac{1}{\frac{1}{4}-\frac{-1}{4}}\int_{-1/4}^{1/4} e^{4x^2}\,dx$

$\frac{1}{\frac{1}{2}}\int = 2\int = 2(.544)=1.0899$

hard

5. Let f be the function defined by $f(x)=e^{x}(ax^2+bx+6)$, where a and b are constants. The function f has a relative extremum at the point $x=-1$, and $f(-1)=17e^{-1}$.

5.4

(a) Find a and b.

5.4

(b) Is the point $(-1,\ 17e^{-1})$ a relative minimum or a relative maximum of f?

(c) Find all the points of inflection of the graph of f.

~~$17e^{-1} = e^{-1}(a(-1)^2+b(-1)+6)$~~

~~$f'(x) = e^x(ax^2+bx+6) + e^x(2ax+b)$~~
~~$0 = e^x(ax^2+bx+6+2ax+b)$~~

~~$17 = a-b+6$~~
~~$13 = -a-b$~~
~~$-13 = a+$~~

since $f(-1)=17e^{-1}$

$f(x)=e^x(ax^2+bx+6)$ can be written

$17e^{-1}=e^{-1}(a(-1)^2+b(-1)+6)$ divide by e^{-1}

$17 = a-b+6$

$13 = a-b$

since $f'(-1)=0$ (max/min) then

$f'(x)=e^x(ax^2+bx+6)+e^x(2ax+b)$ is

$0=e^{-1}(a(-1)^2+b(-1)+6)+e^{-1}(2a(-1)+b)$ $\div e^{-1}$

$0 = a-b+6-2a+b$

$0=-a+6$

$a=6$

$13=6-b$
$-7=b$

$f'(x)=e^x(6x^2+-7x+6)+e^x(2\cdot 6x+-7)$
$=e^x(6x^2+5x-1)$
$e^{-2}(24+-10-1)=+$

inc | MAX | dec
$+$ | | $-$
-2 | -1 | 0

$f''(x)=e^x(6x^2+5x-1)+e^x(12x+5)$
$0=e^x(6x^2+17x+4)$
never
Quad Formula
$x=-.258 + -2.574$

up | down | up
$+$ | $-$ | $+$
-3 -2.5 -1 $-.25$ 0

19. $\int\left(e^{3\ln x}+e^{3x}\right)dx=$

(A) $3+\frac{e^{3x}}{3}+C$ NO

(B) $\frac{x^4}{4}+3e^{3x}+C$

(C) $\frac{e^{x^4}}{4}+3e^{3x}+C$

(D) $\frac{e^{x^4}}{4}+\frac{e^{3x}}{3}+C$

(E) $\frac{x^4}{4}+\frac{e^{3x}}{3}+C$

$\int e^{3x}dx=\frac{1}{3}e^{3x}+C$

cross off

$e^{3\ln x}=e^{\ln x^3}$
$=x^3$

$\int x^3dx$

$\frac{1}{4}x^4$

yes

C

84. What is the x-value of the point at which the tangent line to the graph of $y=e^{x^2}+x$ perpendicular to the line $3y=2x+1$?

5.4

(A) -0.732

(B) -0.589

(C) -0.162

(D) 0.236

(E) 0.361

$\rightarrow y=\frac{2}{3}x+\frac{1}{3}$

$\perp$ to this is slope $-\frac{3}{2}$

$y'=e^{x^2}\cdot 2x+1$

$-\frac{3}{2}=2xe^{x^2}+1$

$x=-.7317$

4.4

31. $\int_0^1 \tan x \, dx =$

$\int_0^1 \frac{\sin x}{\cos x} dx$ $\quad u = \cos x$, $du = -\sin x \, dx$

$\int_0^1 \frac{1}{u}(-du)$

$[\ln|u|]_0^1$

$[\ln|\cos x|]_0^1$

$\ln|\cos 1] - \ln[\cos 0]$

$\ln|\cos 1] - \ln[1]$

$\ln[\cos 1] - 0$

(A) 0

(B) $\frac{\tan^2 1}{2}$

(C) $\ln(\cos(1))$ — needs (−)

(D) $\ln(\sec(1))$ — dec. match.

(E) $\ln(\sec(1)) - 1$

6. An object moves with velocity $v(t) = t^2 - 8t + 7$.

4.1 4.4

(a) Write a polynomial expression for the position of the particle at any time t ≥ 0. $s(t) = \frac{1}{3}t^3 - 4t^2 + 7t + C$

(b) At what time(s) is the particle changing direction? $t = 1 \,\&\, 7$ sec. 3.3

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 4$.

inc dec inc

$0 = t^2 - 8t + 7$

$0 = (t-7)(t-1)$

$t = 7$ $t = 1$

$\int_0^4 (t^2 - 8t + 7) = \int_0^1 v(t)dt + \int_1^4 |v(t)|dt$ 4.4

21.3

$\cancel{[\frac{1}{3}(4)^3 - 4(4)^2 + 7(4)] - [0^3 - 0 + 7(0)]}$

$\cancel{\frac{64}{3} - 64 + 28 - 0}$

$\cancel{21.3 - 36 = -14.6}$ or $\frac{44}{3}$

~~14.6 units in the neg. direction~~

C

44. A radioactive isotope, y, decays according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in seconds. If the half-life of y is 1 minute, then the value of k is 5.6

60 sec.

(A) −41.589 (B) −0.012 (C) 0.027 (D) 0.693 (E) 98.923

$dy = ky \, dt$

$\frac{1}{y} dy = k \, dt$

$\ln y = kt + C$

$y = e^{kt + C} = Ce^{kt}$

$y = Ce^{kt}$

$1 = 2e^{k \cdot 60}$

$\frac{1}{2} = e^{k \cdot 60}$

$\ln(\frac{1}{2}) = 60k$

$-.693 = 60k$

$-.0115 = k$

C

92. A population of bacteria given by $y(t)$ grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in minutes. If $y(10) = 10$ and $y(30) = 25$, what is the value of k? 5.6

(A) −2.079

(B) 0.046

(C) 0.107

(D) 0.125

(E) 0.230

$y = Ce^{kt}$

$10 = Ce^{k \cdot 10}$

$\ln \frac{10}{C} = k \cdot 10$

$\frac{1}{10} \ln(10/c) = k$

$25 = Ce^{k \cdot 30}$

$\ln(\frac{25}{C}) = k \cdot 30$

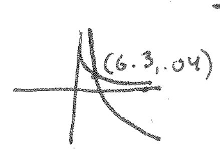
$\frac{1}{30} \ln(25/c) = k$

x y

x y Solve on Calc.

$0 < x < 50$

ZFIT



$y = 6.324e^{.045t}$

c

3. Sea grass grows on a lake. The rate of growth of the grass is $\frac{dG}{dt} = kG$, where k is a constant.

(a) Find an expression for G, the amount of grass in the lake (in tons), in terms of t, the number of years, if the amount of grass is 100 tons initially, and 120 tons after one year.

(b) In how many years will the amount of grass available be 300 tons?

(c) If fish are now introduced into the lake and consume a consistent 80 tons/year of sea grass, how long will it take for the lake to be completely free of sea grass?

now is at the end of year 1

a. $G = Ce^{kt}$

$G = 100e^{kt}$

$120 = 100e^{k\cdot 1}$

$1.2 = e^{k}$

$\ln 1.2 = k$

$.182 = k$

$G = 100e^{.182t}$

b. $300 = 100e^{.182t}$

$3 = e^{.182t}$

$\ln 3 = .182t$

$6.025 = t$ years

~~c. $G = 100e^{.182t} - 80t + 120$~~

~~$0 = 100e^{.182t} - 80t + 120$~~

~~$t = 2.265$ years~~

c. $120e^{.182t} - 80t = 0$

$t = 2.265$ years

★Double Checked

c

2. At time $t = 0$ minutes, the temperature of a cup of coffee is 180 degrees Fahrenheit. Left in a room whose temperature is 70 degrees Fahrenheit, the coffee cools so that its temperature function $T(t)$, also measured in degrees Fahrenheit, satisfies the differential equation $\frac{dT}{dt} = -\frac{1}{2}T + 35$.

(a) Find an expression for $T(t)$ using the initial condition $T(0) = 180$.

(b) Find $\lim_{t \to \infty} T(t)$. Explain what this limit means in the context of the problem.

(c) At what time t is the temperature of the coffee decreasing at the rate of 15 degrees Fahrenheit per minute? How hot is the coffee at that point? Indicate units of measurement.

a. $dT = (-\frac{1}{2}T + 35)dt$

$\int \frac{1}{-\frac{1}{2}T+35}\,dT = \int 1\,dt$

$-2\ln|-\frac{1}{2}T+35| = t + c$

$\ln|-\frac{1}{2}T+35| = -\frac{1}{2}t + c$

$-\frac{1}{2}T + 35 = e^{-\frac{1}{2}t + c}$

$180 = Ce^{0} + 70$

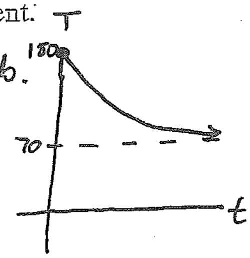
$110 = C$

$-\frac{1}{2}T = Ce^{-\frac{1}{2}t} - 35$

$T(t) = Ce^{-\frac{1}{2}t} + 70$

$T(t) = \cancel{180}\,110\, e^{-\frac{1}{2}t} + 70$

b.



decay model

$\lim_{t \to \infty} T(t) = 70$

c. $\frac{dT}{dt}$

$-15 = -\frac{1}{2}T + 35$

$-50 = -\frac{1}{2}T$

$100 = T$ Temp = 100°F

$100 = \cancel{180}\,110\, e^{-\frac{1}{2}t} + 70$

$30 = \cancel{180}\,110\, e^{-\frac{1}{2}t}$

$\frac{3}{11} = e^{-\frac{1}{2}t}$

$\ln(3/11) = -\frac{1}{2}t$

$-2\ln(3/11) = t$

2.598 min $= t$ time.