

## Lesson 562 Differential Equations proportional relationships

### Types of Variation

k is called the Constant of Variation

y varies directly as x means  $y = k \cdot x$

y varies indirectly as x means  $y = k / x$

z varies jointly as x and y means  $z = k \cdot x \cdot y$

### Proportional Rates of Change

In a situation where the rate of change of one variable is proportional to the rate of change of another:

$$y' = ky$$

Then  $y = Ce^{kt}$  where C is the initial value and k is the proportionality constant.

**Note:** if k is positive, exponential growth will occur  
if k is negative then exponential decay will occur.

### Examples

**In Exercises 11–14, write and solve the differential equation that models the verbal statement.**

11. The rate of change of  $Q$  with respect to  $t$  is inversely proportional to the square of  $t$ .
12. The rate of change of  $P$  with respect to  $t$  is proportional to  $10 - t$ .

In Exercises 17–20, find the function  $y = f(t)$  passing through the point  $(0, 10)$  with the given first derivative. Use a graphing utility to graph the solution.

17.  $\frac{dy}{dt} = \frac{1}{2}t$

18.  $\frac{dy}{dt} = -\frac{3}{4}\sqrt{t}$

19.  $\frac{dy}{dt} = -\frac{1}{2}y$

20.  $\frac{dy}{dt} = \frac{3}{4}y$

In Exercises 21–24, write and solve the differential equation that models the verbal statement. Evaluate the solution at the specified value of the independent variable.

21. The rate of change of  $y$  is proportional to  $y$ . When  $x = 0$ ,  $y = 4$  and when  $x = 3$ ,  $y = 10$ . What is the value of  $y$  when  $x = 6$ ?
22. The rate of change of  $N$  is proportional to  $N$ . When  $t = 0$ ,  $N = 250$  and when  $t = 1$ ,  $N = 400$ . What is the value of  $N$  when  $t = 4$ ?

In Exercises 31–42, use integration to find a general solution of the differential equation.

31.  $\frac{dy}{dx} = 3x^2$

32.  $\frac{dy}{dx} = x^3 - 4x$

33.  $\frac{dy}{dx} = \frac{x}{1+x^2}$

34.  $\frac{dy}{dx} = \frac{e^x}{1+e^x}$

35.  $\frac{dy}{dx} = \frac{x-2}{x}$

36.  $\frac{dy}{dx} = x \cos x^2$

37.  $\frac{dy}{dx} = \sin 2x$

38.  $\frac{dy}{dx} = \tan^2 x$

39.  $\frac{dy}{dx} = x\sqrt{x-3}$

40.  $\frac{dy}{dx} = x\sqrt{5-x}$

41.  $\frac{dy}{dx} = xe^{x^2}$

42.  $\frac{dy}{dx} = 5e^{-x/2}$