Assignment 554

In Exercises 41–56, find the derivative of the function.

42. $g(x) = 2^{-x}$

44. $y = x(6^{-2x})$

46. $f(t) = \dfrac{3^{2t}}{t}$

48. $g(\alpha) = 5^{-\alpha/2} \sin 2\alpha$

50. $y = \log_{10} 2x$

52. $h(x) = \log_3 \dfrac{x\sqrt{x-1}}{2}$

54. $y = \log_{10} \dfrac{x^2 - 1}{x}$

56. $f(t) = t^{3/2} \log_2 \sqrt{t+1}$

59. $y = (x - 2)^{x+1}$

60. $y = (1 + x)^{1/x}$

78. ***Depreciation*** After t years, the value of a car purchased for \$20,000 is

$$V(t) = 20{,}000\left(\tfrac{3}{4}\right)^t.$$

(a) Use a graphing utility to graph the function and determine the value of the car 2 years after it was purchased.

(b) Find the rate of change of V with respect to t when $t = 1$ and $t = 4$.

(c) Use a graphing utility to graph $V'(t)$ and determine the horizontal asymptote of $V'(t)$. Interpret its meaning in the context of the problem.

In Exercises 61–68, find or evaluate the integral.

62. $\int 5^{-x}\,dx$

64. $\int_{-2}^{0} (3^3 - 5^2)\,dx$

66. $\int (3 - x)7^{(3-x)^2}\,dx$

68. $\int 2^{\sin x} \cos x\,dx$

121. ***Probability*** A car battery has an average lifetime of 48 months with a standard deviation of 6 months. The battery lives are normally distributed. The probability that a given battery will last between 48 months and 60 months is

$$0.0665 \int_{48}^{60} e^{-0.0139(t-48)^2}\,dt.$$

Use the integration capabilities of a graphing utility to approximate the integral. Interpret the resulting probability.