Lesson 531 Derivatives of Inverse Functions

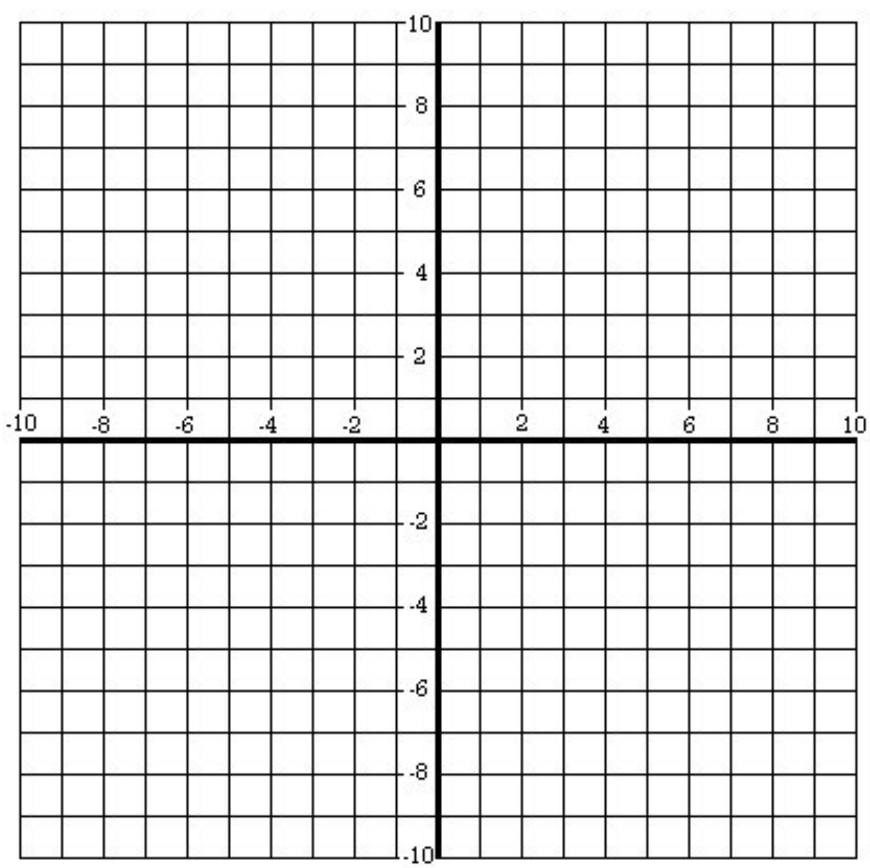
Part 1 $f(x) = x^2 - 4$ find the inverse of f(x), which we will call g(x)

g(x) = ____________________

Use your calculator to fill in the table.

x values	Ordered pair on f(x)	Slopes on f(x) at these points	Points that MUST exist on g(x)	Slopes at these points
1				
2				
3				

Sketch f(x), g(x), and y = x on the grid:



Summary:

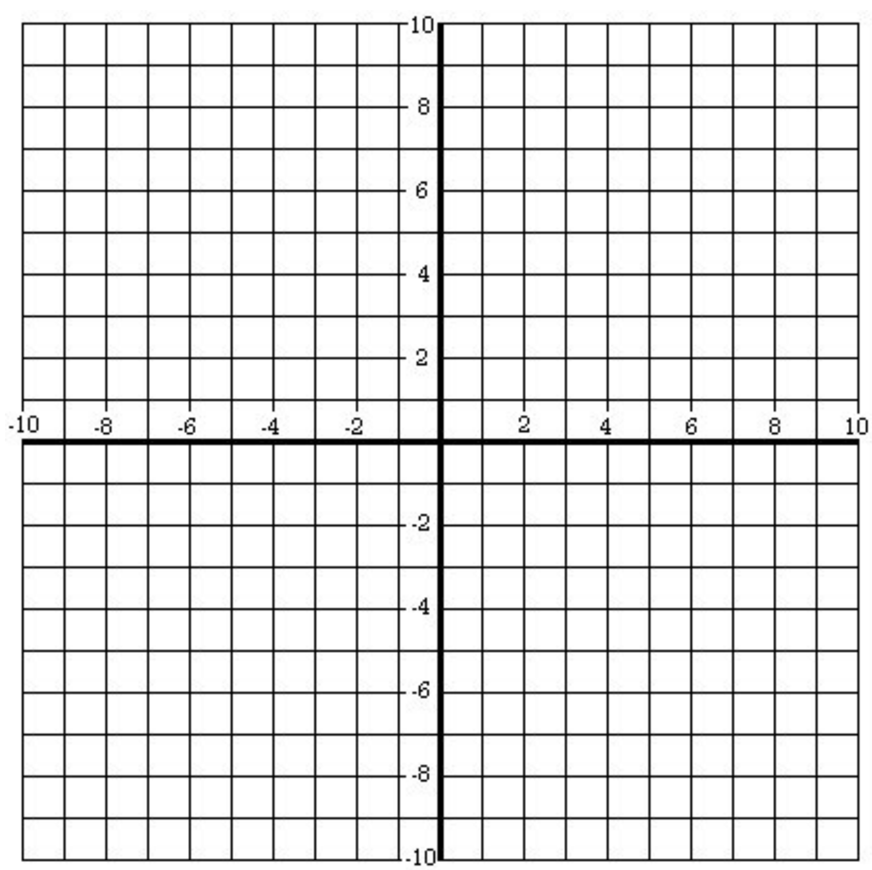
Part 2 $f(x) = \ln(x + 3)$ find the inverse of $f(x)$, which we will call $g(x)$

$g(x) =$ ____________________

Use your calculator to fill in the table.

x values	Ordered pair on f(x)	Slopes on f(x) at these points	Points that MUST exist on g(x)	Slopes at these points
-2				
-1	Hint: use store command Y→A		Hint: use stored value A	Hint: use stored value A
1	Hint: use store command Y→B		Hint: use stored value B	Hint: use stored value B

Sketch $f(x)$, $g(x)$, and $y = x$ on the grid:



Summary:

Book Definition for the Derivatives of Inverses

Function Name	**f(x)**	**f'(x)**
Inverses	f(x) and g(x) are inverse functions where $f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$	$g'(x) = \frac{1}{f'(g(x))}$ where the bottom is not zero

Extra Terms:

One-to-one: passes the vertical line test and horizontal line test. Required for inverse to exist.

Monotonic: always increase on the entire function or always decreasing on the entire function.

Examples

1. g(x) is the inverse of f(x). f(2) = 4 and f'(2) = -½ the what g'(4)?

2. $f^{-1}(x)$ is the inverse of f(x). f(1) = 3 and f'(1) = 5. What tangent line must exist along $f^{-1}(x)$?

Find $(f^{-1})'(x)$

1) $f(x) = -3x + 3$

2) $f(x) = -2x + 3$

Find $(f^{-1})'(2)$

7) $f(x) = x^7 + x - 3$

8) $f(x) = 3x^5 + 2x + 5$

Assignment 531

Find $(f^{-1})'(x)$ for 3 and 4

3) $f(x) = -5x + 1$

4) $f(x) = -2x + 2$

$f(x) = 2x - 4$

$f(x) = 3 + \sqrt{(x)}$

$f(x) = (x - 2)^2$

$f(x) = \ln(x)$

Find $(f^{-1})'(1)$ for 5 and 6

5) $f(x) = \sqrt{-2x - 3}$

6) $f(x) = -4x^3 - 4$

$f(x) = x^3 - 2$ find $(f^{-1})'(6)$

$f(x) = \sqrt{x^2 - x + 2}$ find $(f^{-1})'(2)$

$f(x) = \sin x$ find $(f^{-1})'(1)$

$f(x) = e^3$ find $(f^{-1})'(e)$

Function f(x) is described in the following table:

x	f(x)	f'(x)
0	2	-4
1	0	½
2	1	5

Part 1:

find $(f^{-1})'(0)$

find $(f^{-1})'(1)$

find $(f^{-1})'(2)$

Part 2:

Approximate the average value of f(x) on the interval [0, 2]

Approximate the average value of f'(x) on the interval [0, 2]

45. ***Cost*** Suppose you need 50 pounds of two commodities costing \$1.25 and \$1.60 per pound.

(a) Verify that the total cost is

$$y = 1.25x + 1.60(50 - x)$$

where x is the number of pounds of the less expensive commodity.

(b) Find the inverse function of the cost function. What does each variable represent in the inverse function?

(c) Use the context of the problem to determine the domain of the inverse function.

(d) Determine the number of pounds of the less expensive commodity purchased if the total cost is \$73.

Extra Problems

In Exercises 71–76, find $(f^{-1})'(a)$ for the function f and real number a.

Function	*Real Number*
71. $f(x) = x^3 + 2x - 1$	$a = 2$
72. $f(x) = \frac{1}{27}(x^5 + 2x^3)$,	$a = -11$
73. $f(x) = \sin x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$a = \frac{1}{2}$
74. $f(x) = \cos 2x, \quad 0 \le x \le \frac{\pi}{2}$	$a = 1$
75. $f(x) = x^3 - \frac{4}{x}$	$a = 6$
76. $f(x) = \sqrt{x - 4}$	$a = 2$

$y = x^2 - 4$

Lesson 531 Derivatives of Inverse Functions

$x = y^2 - 4$

$x + 4 = y^2$

$\pm\sqrt{x+4} = y$

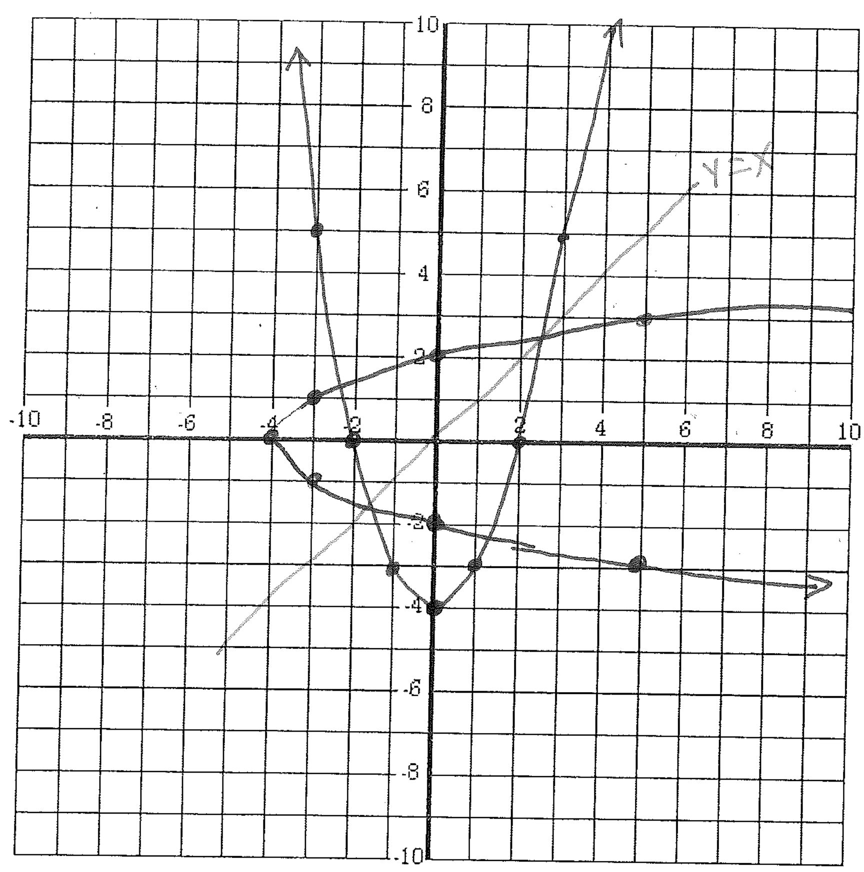
Part 1 $f(x) = x^2 - 4$ find the inverse of f(x), which we will call g(x)

g(x) = $\pm\sqrt{x+4}$

Use your calculator to fill in the table.

x values	Ordered pair on f(x)	Slopes on f(x) at these points	Points that MUST exist on g(x)	Slopes at these points
1	(1, −3)	2	(−3, 1)	$\frac{1}{2}$
2	(2, 0)	4	(0, 2)	$\frac{1}{4}$
3	(3, 5)	6	(5, 3)	$\frac{1}{6}$

Sketch f(x), g(x), and y = x on the grid:



$f'(x) = 2x$

$f'(1) = 2$

$f'(2) = 4$

$f'(3) = 6$

$g'(x) = \frac{1}{2}(x+4)^{-1/2}$

$g'(x) = \frac{1}{2\sqrt{x+4}}$

$g'(-3) = \frac{1}{2\sqrt{-3+4}} = \frac{1}{2}$

$g'(0) = \frac{1}{2\sqrt{0+4}} = \frac{1}{4}$

$g'(5) = \frac{1}{2\sqrt{5+4}} = \frac{1}{6}$

Summary: Original (x,y) becomes (y,x)

Slopes $f'(x) = \frac{a}{b}$ $g'(y) = \frac{b}{a}$

Part 2 $f(x) = \ln(x+3)$ find the inverse of f(x), which we will call g(x)

$y = \ln(x+3)$

$x = \ln(y+3)$

$e^x = y+3$

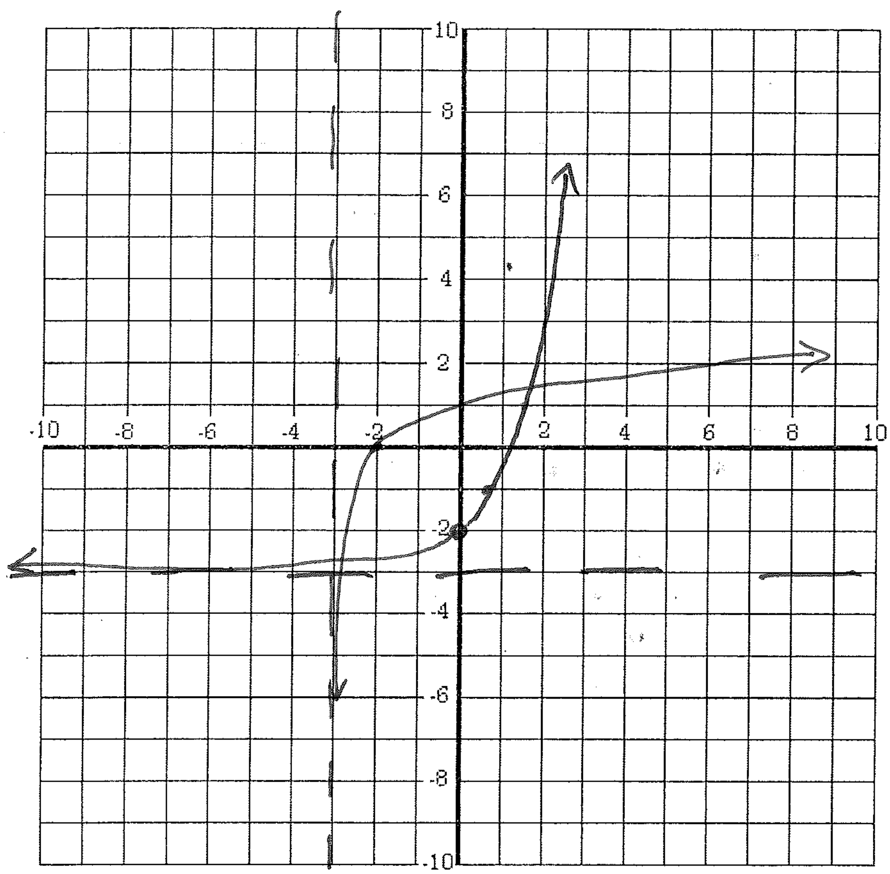
$e^x - 3 = y$

g(x) = $e^x - 3$

Use your calculator to fill in the table.

x values	Ordered pair on f(x)	Slopes on f(x) at these points	Points that MUST exist on g(x)	Slopes at these points
-2	(-2, 0)	1	(0, -2)	1
-1	(-1, .69315) Hint: use store command Y→A	$\frac{1}{2}$	(.69315, -1) Hint: use stored value A	2 Hint: use stored value A
1	(1, 1.3863) Hint: use store command Y→B	$\frac{1}{4}$	(1.3863, 1) Hint: use stored value B	4 Hint: use stored value B

Sketch f(x), g(x), and y = x on the grid:



$f'(x) = \frac{1}{x+3}$

$f'(-2) = 1$

$f'(-1) = \frac{1}{2}$

$f'(1) = \frac{1}{4}$

$g'(x) = e^x$

$g'(0) = 1$

$g'(.69315) = e^{.69315} = 2$

$g'(1.3863) = 4$

Summary:

Book Definition for the Derivatives of Inverses

Function Name	f(x)	f'(x)
Inverses	f(x) and g(x) are inverse functions where $f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$	$g'(x) = \frac{1}{f'(g(x))}$ where the bottom is not zero

Extra Terms:

One-to-one: passes the vertical line test and horizontal line test. Required for inverse to exist.

Monotonic: always increase on the entire function or always decreasing on the entire function.

Examples

1. g(x) is the inverse of f(x). f(2) = 4 and f'(2) = -½ the what g'(4)?

	f	g
Orig	(2,4)	(4,2)
Slopes	$-\frac{1}{2}$	-2

(-2)

2. $f^{-1}(x)$ is the inverse of f(x). f(1) = 3 and f'(1) = 5. What tangent line must exist along $f^{-1}(x)$?

	f	g
Orig	(1,3)	(3,1)
Slope	5	$\frac{1}{5}$

$y - 1 = \frac{1}{5}(x-3)$

Find $(f^{-1})'(x)$

1) $f(x) = -3x + 3$

$y = -3x+3$

$x = -3y+3$

$x - 3 = -3y$

$y = \frac{x-3}{-3} = -\frac{1}{3}x + 1$

2) $f(x) = -2x + 3$

Find $(f^{-1})'(2)$

7) $f(x) = x^7 + x - 3$

8) $f(x) = 3x^5 + 2x + 5$

	f	g
Orig	(1.2, 2)	(2, 1.2)
Slope	27.9335	

Assignment 531

Find $(f^{-1})'(x)$ for 3 and 4

3) $f(x) = -5x + 1$

4) $f(x) = -2x + 2$

f(x) = 2x – 4

f(x) = 3 + √(x)

$f(x) = (x - 2)^2$

f(x) = ln (x)

Find $(f^{-1})'(1)$ for 5 and 6

5) $f(x) = \sqrt{-2x-3}$

	f	f^{-1}
point	(-2,1)	(1,-2)
slope	-1	(-1)

$1 = \sqrt{-2x-3}$
$1 = -2x-3$
$4 = -2x$
$-2 = x$
$f'(x) = \frac{1}{2}(-2x-3)^{-1/2}(-2)$
$f'(-2) = \frac{-1}{\sqrt{-2(-2)-3}} = -1$

6) $f(x) = -4x^3 - 4$

	f	f^{-1}
point	(-1.077, 1)	(1, -1.077)
Slope:	-13.925	-.072

$f(x) = x^3 - 2$ find $(f^{-1})'(6)$

$f(x) = \sqrt{x^2 - x + 2}$ find $(f^{-1})'(2)$

(-1, 2) (2, -1)
$-\frac{3}{4}$ $-\frac{4}{3}$

(2, 2) (2, 2)
$\frac{3}{4}$ $\frac{4}{3}$

f(x) = sin x find $(f^{-1})'(1)$

	f	f^{-1}
point	$(\frac{\pi}{2}, 1)$	$(1, \frac{\pi}{2})$
slope	0	undef

~~$f(x) = e^3$ find $(f^{-1})'(e)$~~

Function f(x) is described in the following table:

x	f(x)	f'(x)
0	2	-4
1	0	½
2	1	5

Part 1:

find $(f^{-1})'(0)$

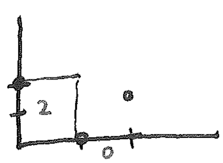
find $(f^{-1})'(1)$

find $(f^{-1})'(2)$

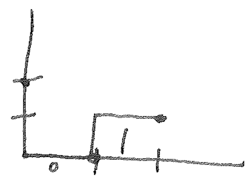
f $\quad$ f^{-1}
pt. (1, 0) (0, 1)
Slope $\frac{1}{2}$ $\quad$ **2**

Part 2:

Approximate the average value of f(x) on the interval [0, 2]



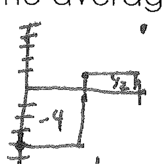
Left: 2



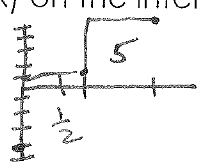
Right: 1

Ave = 1.5

Approximate the average value of f'(x) on the interval [0, 2]



L = -3.5



R = 5.5

Ave: $\frac{-3.5+5.5}{2} = 1$

45. *Cost* Suppose you need 50 pounds of two commodities costing \$1.25 and \$1.60 per pound.

(a) Verify that the total cost is

$y = 1.25x + 1.60(50 - x)$

where x is the number of pounds of the less expensive commodity.

x = lbs of type 1
verified

(b) Find the inverse function of the cost function. What does each variable represent in the inverse function?

(c) Use the context of the problem to determine the domain of the inverse function. D: $62.5 \le x \le 80$

(d) Determine the number of pounds of the less expensive commodity purchased if the total cost is \$73.

Domain: $0 \le x \le 80$
Range: $62.5 \le y \le 80$

$y = 1.25x + 80 - 1.6x$
$y = 80 - .35x$
Total cost = 80 − .35(lbs of type 1)
Inverse: $x = 80 - .35y$
$\frac{x - 80}{-.35} = y$
$\frac{(\text{Total cost} - 80)}{-.35}$ = lbs of type 1

$y = 80 - .35x$
$73 = 80 - .35x$
$-7 = -.35x$
$20 = x$
cheaper

20 lbs

$30 = y$
more expensive

Extra Problems

In Exercises 71–76, find $(f^{-1})'(a)$ for the function f and real number a.

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74. $f(x) = \cos 2x, \quad 0 \le x \le \frac{\pi}{2}$	$a = 1$
75. $f(x) = x^3 - \frac{4}{x}$	$a = 6$
76. $f(x) = \sqrt{x - 4}$	$a = 2$