

Lesson 521 Integration with e^x and the Natural Log Functions

Derivative Review

Function Name	f(x)	f'(x)
Natural Log	$f(x) = \ln(g(x))$	$f'(x) = \frac{g'(x)}{g(x)}$

Function Name	f(x)	f'(x)
e^x	$f(x) = e^{g(x)}$	$f'(x) = g'(x) \cdot e^{g(x)}$

Antiderivative using natural log:

Note the use of absolute value so the domain of $1/x$ and $\ln x$ are the same.
Domain = all reals but zero

$$\int \frac{1}{x} dx = \ln |x| + C$$

When using the substitution method, u will closely resemble the denominator

Antiderivative using e^x :

$$\int e^x dx = e^x + C$$

When using the substitution method, u will closely resemble the power of e

Interesting Connection

The natural log can be defined: $\ln x = \int_1^x \frac{1}{t} dt$ for $x > 0$ so $\ln e = \int_1^e \frac{1}{t} dt = 1$

Examples:

Evaluate each indefinite integral. Use the provided substitution.

1) $\int \frac{20x^4}{4x^5 + 3} dx; u = 4x^5 + 3$

2) $\int 36x^2 e^{4x^3 + 3} dx; u = 4x^3 + 3$

3) $\int 80x^3 \cdot 3^{5x^4 - 2} dx; u = 5x^4 - 2$

4) $\int \frac{2}{x(-1 + \ln 4x)} dx; u = -1 + \ln 4x$

Evaluate each indefinite integral.

5) $\int \frac{12x^2}{x^3 + 2} dx$

6) $\int \frac{20e^{5x}}{e^{5x} + 3} dx$

7) $\int 10 \sin -2x \cdot e^{\cos -2x} dx$

8) $\int \frac{5e^{-3 + \ln 3x}}{x} dx$

Assignment 521

Integrate the following

1. $\int \frac{5}{x} dx$
2. $\int \frac{10}{x} dx$
3. $\int \frac{1}{x+1} dx$
4. $\int \frac{1}{x-5} dx$
5. $\int \frac{1}{3-2x} dx$
6. $\int \frac{1}{3x+2} dx$
7. $\int \frac{x}{x^2+1} dx$
8. $\int \frac{x^2}{3-x^3} dx$
9. $\int \frac{x^2-4}{x} dx$
10. $\int \frac{x}{\sqrt{9-x^2}} dx$

Challenges:

25. $\int \frac{1}{1+\sqrt{2x}} dx$
27. $\int \frac{\sqrt{x}}{\sqrt{x}-3} dx$

$$87. \int e^{5x}(5) dx$$

$$89. \int_0^1 e^{-2x} dx$$

$$91. \int xe^{-x^2} dx$$

$$93. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$95. \int \frac{e^{-x}}{1 + e^{-x}} dx$$

$$97. \int_1^3 \frac{e^{3/x}}{x^2} dx$$

$$99. \int e^x \sqrt{1 - e^x} dx$$

$$101. \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$$

$$103. \int \frac{5 - e^x}{e^{2x}} dx$$

$$105. \int e^{\sin \pi x} \cos \pi x dx$$

$$107. \int e^{-x} \tan(e^{-x}) dx$$

Assignment 522

19. $\int \frac{(\ln x)^2}{x} dx$

21. $\int \frac{1}{\sqrt{x+1}} dx$

23. $\int \frac{2x}{(x-1)^2} dx$

29. $\int \frac{\cos \theta}{\sin \theta} d\theta$

31. $\int \csc 2x dx$

33. $\int \frac{\cos t}{1 + \sin t} dt$

35. $\int \frac{\sec x \tan x}{\sec x - 1} dx$

30. $\int \tan 5\theta d\theta$

32. $\int \sec \frac{x}{2} dx$

34. $\int \frac{\csc^2 t}{\cot t} dt$

36. $\int (\sec t + \tan t) dt$

43. $\int_0^4 \frac{5}{3x+1} dx$

45. $\int_1^e \frac{(1 + \ln x)^2}{x} dx$

47. $\int_0^2 \frac{x^2 - 2}{x+1} dx$

49. $\int_1^2 \frac{1 - \cos \theta}{\theta - \sin \theta} d\theta$

$$88. \int e^{-x^4}(-4x^3) dx$$

$$90. \int_3^4 e^{3-x} dx$$

$$92. \int x^2 e^{x^3/2} dx$$

$$94. \int \frac{e^{1/x^2}}{x^3} dx$$

$$96. \int \frac{e^{2x}}{1 + e^{2x}} dx$$

$$98. \int_0^{\sqrt{2}} x e^{-(x^2/2)} dx$$

$$100. \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$102. \int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx$$

$$104. \int \frac{e^{2x} + 2e^x + 1}{e^x} dx$$

$$106. \int e^{\sec 2x} \sec 2x \tan 2x dx$$

$$108. \int \ln(e^{2x-1}) dx$$

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Key

Derivative Review

Function Name	$f(x)$	$f'(x)$
Natural Log	$f(x) = \ln(g(x))$	$f'(x) = \frac{g'(x)}{g(x)}$

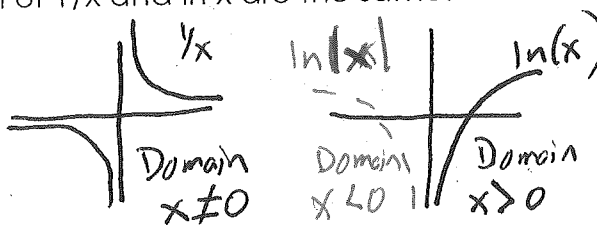
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Antiderivative using natural log:

Note the use of absolute value so the domain of $1/x$ and $\ln x$ are the same.

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Examples:

Evaluate each indefinite integral. Use the provided substitution.

$$1) \int \frac{20x^4}{4x^5+3} dx; u=4x^5+3$$

$$du=20x^4 dx$$

$$\int \frac{1}{u} du$$

$$\ln|u|$$

$$\ln|4x^5+3| + C$$

try $\ln|4x^5+3|$
 $\frac{20x^4}{4x^5+3}$

$$2) \int 36x^2 e^{4x^3+3} dx; u=4x^3+3$$

$$du=12x^2 dx$$

$$3du=36x^2 dx$$

$$3 \int e^u du$$

$$3e^u \rightarrow 3e^{4x^3+3} + C$$

$$3) \int 80x^3 \cdot e^{5x^4-2} dx; u=5x^4-2$$

$$du=20x^3 dx$$

$$4du=80x^3 dx$$

$$4 \int e^u du$$

$$4e^u \rightarrow 4e^{5x^4-2} + C$$

$$4) \int \frac{2}{x(-1+\ln 4x)} dx; u=-1+\ln 4x$$

$$du=\frac{4}{4x} dx$$

$$2 \int \frac{1}{u} du$$

$$2du=\frac{2}{x} dx$$

$$2 \ln|u| \rightarrow 2 \ln|-1+\ln 4x| + C$$

Evaluate each indefinite integral.

$$5) \int \frac{12x^2}{x^3+2} dx$$

$$u=x^3+2$$

$$du=3x^2 dx$$

$$4du=12x^2 dx$$

$$4 \int \frac{1}{u} du$$

$$4 \ln|u|$$

$$4 \ln|x^3+2| + C$$

$$6) \int \frac{20e^{5x}}{e^{5x}+3} dx$$

$$u=e^{5x}+3$$

$$du=5e^{5x} dx$$

$$4du=20e^{5x} dx$$

$$4 \int \frac{1}{u} du$$

$$4 \ln|e^{5x}+3| + C$$

$$7) \int 10 \sin(-2x) \cdot e^{\cos(-2x)} dx$$

$$u=\cos(-2x)$$

$$du=-\sin(-2x)(-2) dx$$

$$5du=10 \sin(-2x) dx$$

$$5 \int e^u du$$

$$5e^u$$

$$5e^{\cos(-2x)} + C$$

$$8) \int \frac{5e^{-3+\ln 3x}}{x} dx = 5 \int e^u du$$

$$u=-3+\ln(3x)$$

$$du=\frac{3}{3x} dx$$

$$5du=\frac{5}{x} dx$$

$$5e^u$$

$$5e^{-3+\ln(3x)}$$

$$5e^{-3+\ln(3x)} + C$$

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10. $\int \frac{x}{\sqrt{9-x^2}} dx$

25. $\int \frac{1}{1+\sqrt{2x}} dx$

27. $\int \frac{\sqrt{x}}{\sqrt{x}-3} dx$

(25) $\int \left[\frac{1}{1+\sqrt{2x}} \right] \left[\frac{\sqrt{2x}}{\sqrt{2x}} \right] dx$

$u = 1 + \sqrt{2x} \quad \sqrt{2x} = u - 1$

$du = \frac{1}{\sqrt{2x}} dx$

$\int \frac{u-1}{u} du = \int \left(1 - \frac{1}{u}\right) du = u - \ln|u|$
 $= \sqrt{2x} - \ln|1+\sqrt{2x}| + C$

(27) $\int \frac{\sqrt{x}}{\sqrt{x}-3} \left[\frac{\sqrt{x}}{\sqrt{x}} \right] dx = \int \frac{x}{(\sqrt{x}-3)\sqrt{x}} dx$

$u = \sqrt{x} - 3 \quad du = \frac{1}{2\sqrt{x}} dx \quad 2 du = \frac{dx}{\sqrt{x}}$

$u+3 = \sqrt{x} \quad x = (u+3)^2$

$2 \int \frac{(u+3)^2}{u} du = 2 \int \left(u + 6 + \frac{9}{u}\right) du$

$u^2 + 12u + 18 \ln|u| = (\sqrt{x}-3)^2 + 12(\sqrt{x}-3) + 18 \ln|\sqrt{x}-3| + C$

(1) $5 \int \frac{1}{x} dx = 5 \ln|x| + C$

(3) $\ln|x+1| + C$

(5) $u = 3 - 2x \quad du = -2dx$
 $-\frac{1}{2} \int \frac{1}{u} du = -\frac{1}{2} \ln|u| = -\frac{1}{2} \ln|3-2x| + C$

(7) $u = x^2 + 1 \quad du = 2x dx$
 $\frac{1}{2} du = x dx$
 $\frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| = \frac{1}{2} \ln|x^2+1| + C$

(9) $\int \frac{x^2}{x} - \frac{4}{x} dx = \int \left(x - \frac{4}{x}\right) dx = \frac{1}{2} x^2 - 4 \ln|x| + C$

(10) $\int x(9-x^2)^{-1/2} dx = -\frac{1}{2} \int u^{-1/2} du = \frac{-1}{\frac{1}{2}} (u)^{1/2} = -\sqrt{9-x^2} + C$

$u = 9 - x^2$
 $du = -2x dx$
 $-\frac{1}{2} du = x dx$

$$87. \int e^{5x} (5) dx \quad \begin{array}{l} u = 5x \\ du = 5dx \end{array} \quad \int e^u du = e^u = e^{5x} + C$$

$$89. \int_0^1 e^{-2x} dx \quad \begin{array}{l} u = -2x \\ du = -2dx \\ -\frac{1}{2} du = dx \end{array} \quad -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u \Big|_0^1 = -\frac{1}{2} e^{-2} - \left(-\frac{1}{2} e^0\right) = 1 - \frac{1}{2} e^{-2}$$

$$91. \int x e^{-x^2} dx \quad \begin{array}{l} u = -x^2 \\ du = -2x dx \\ -\frac{1}{2} du = x dx \end{array} \quad -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u = \frac{1}{2} e^{-x^2} + C$$

$$93. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx \quad \begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2} x^{-1/2} dx \\ 2 du = \frac{1}{\sqrt{x}} dx \end{array} \quad 2 \int e^u du = 2e^u = 2^{\sqrt{x}} + C$$

$$95. \int \frac{e^{-x}}{1+e^{-x}} dx \quad \begin{array}{l} u = 1+e^{-x} \\ du = -e^{-x} dx \end{array} \quad - \int \frac{1}{u} du = -\ln|u| = -\ln|1+e^{-x}| + C$$

$$97. \int_1^3 \frac{e^{3/x}}{x^2} dx \quad \begin{array}{l} u = 3x^{-1} \\ du = -3x^{-2} dx \\ -\frac{1}{3} du = x^{-2} dx \end{array} \quad -\frac{1}{3} \int e^u du = -\frac{1}{3} e^u \Big|_1^3 = -\frac{1}{3} e^{3/3} + \frac{1}{3} e^{3/1}$$

$$99. \int e^x \sqrt{1-e^x} dx \quad \begin{array}{l} u = 1-e^x \\ du = -e^x dx \end{array} \quad - \int u^{1/2} du = -\frac{2}{3} u^{3/2} = \frac{2}{3} (1-e^x)^{3/2} + C$$

$$101. \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx \quad \begin{array}{l} u = e^x - e^{-x} \\ du = e^x + e^{-x} dx \end{array} \quad \int \frac{1}{u} du = \ln|u| = \ln|e^x - e^{-x}| + C$$

$$103. \int \frac{5-e^x}{e^{2x}} dx \quad \rightarrow \int (5e^{-2x} - e^{-x}) dx \quad \begin{array}{l} u = -2x \\ du = -2dx \\ -\frac{1}{2} du = dx \end{array} \quad \left. \begin{array}{l} -\frac{5}{2} \int e^u du + \int e^{-x} dx \\ -\frac{5}{2} e^{-2x} + e^{-x} + C \end{array} \right\}$$

$$105. \int e^{\sin \pi x} \cos \pi x dx$$

$$107. \int e^{-x} \tan(e^{-x}) dx \quad \begin{array}{l} u = \sin(\pi x) \\ du = \cos(\pi x) \pi dx \\ \frac{1}{\pi} du = \cos(\pi x) dx \end{array} \quad \left. \begin{array}{l} \frac{1}{\pi} \int e^u du = \frac{1}{\pi} e^{\sin(\pi x)} + C \end{array} \right\}$$

$$u = e^{-x} \\ du = -e^{-x} dx$$

$$- \int \tan(u) du$$

$$- \int \frac{\sin(u)}{\cos(u)} du = \ln|\cos(u)| \\ = \ln|\cos(e^{-x})| + C$$

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11. $\int \frac{x^2 + 2x + 3}{x^3 + 3x^2 + 9x} dx$

13. $\int \frac{x^2 - 3x + 2}{x + 1} dx$

15. $\int \frac{x^3 - 3x^2 + 5}{x - 3} dx$

17. $\int \frac{x^4 + x - 4}{x^2 + 2} dx$

19. $\int \frac{(\ln x)^2}{x} dx$ $u = \ln x$ $du = \frac{1}{x} dx$ $\int u^2 du = \frac{1}{3} u^3 = \frac{1}{3} (\ln x)^3 + C$

21. $\int \frac{1}{\sqrt{x+1}} dx = 2\sqrt{x+1} + C$

23. $\int \frac{2x}{(x-1)^2} dx$ $u = x-1$ $x = u+1$ $du = dx$ $2 \int \frac{u+1}{u^2} du = 2 \int (\frac{1}{u} + u^{-2}) du = 2 \ln|u| - u^{-1} + C$
 $2 \ln|x-1| - \frac{1}{x-1} + C$

26. $\int \frac{1}{1 + \sqrt{3x}} dx = \int \frac{1}{1 + \sqrt{3x}} \left(\frac{\sqrt{3x}}{\sqrt{3x}}\right) dx$ $u = 1 + \sqrt{3x}$
 $u - 1 = \sqrt{3x}$
 $du = \frac{\sqrt{3}}{2\sqrt{x}} dx$ $\frac{2}{\sqrt{3}} du = \frac{1}{\sqrt{3x}} dx$ $\frac{2}{3} \int \frac{u-1}{u} du = \frac{2}{3} \int (1 - \frac{1}{u}) du$
 $\frac{2}{3} u - \frac{2}{3} \ln|u| = \frac{2}{3} (1 + \sqrt{3x}) - \frac{2}{3} \ln|1 + \sqrt{3x}| + C$

29. $\int \frac{\cos \theta}{\sin \theta} d\theta$ $u = \sin \theta$ $du = \cos \theta d\theta$ $\int \frac{1}{u} du = \ln|u| + C$

31. $\int \csc 2x dx = \int \frac{1}{2 \sin(x) \cos(x)} dx$

33. $\int \frac{\cos t}{1 + \sin t} dt = \ln|1 + \sin t| + C$

35. $\int \frac{\sec x \tan x}{\sec x - 1} dx = \ln|\sec x - 1| + C$

$$88. \int e^{-x^4}(-4x^3) dx = e^{-x^4} + C$$

$$90. \int_3^4 e^{3-x} dx = -e^{3-x} + C$$

$$92. \int x^2 e^{x^3/2} dx \quad \left. \begin{array}{l} u = \frac{1}{2}x^3 \\ du = \frac{3}{2}x^2 dx \\ \frac{2}{3}du = x^2 dx \end{array} \right\} \frac{2}{3} \int e^u du = \frac{2}{3} e^{\frac{1}{2}x^3} + C$$

$$94. \int \frac{e^{1/x^2}}{x^3} dx \quad \left. \begin{array}{l} u = x^{-2} \\ du = -2x^{-3} dx \\ \frac{1}{2}du = x^{-3} dx \end{array} \right\} -\frac{1}{2} \int e^u du = -\frac{1}{2} e^{1/x^2} + C$$

$$96. \int \frac{e^{2x}}{1+e^{2x}} dx \quad \left. \begin{array}{l} u = 1+e^{2x} \\ du = e^{2x}(2) dx \\ \frac{1}{2}du = e^{2x} dx \end{array} \right\} \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|1+e^{2x}| + C$$

$$98. \int_0^{\sqrt{2}} x e^{-(x^2/2)} dx \quad \left. \begin{array}{l} u = -\frac{1}{2}x^2 \\ du = -x dx \end{array} \right\} - \int e^u du = -e^{-\frac{1}{2}x^2} + C$$

$$100. \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx \quad \left. \begin{array}{l} u = e^x + e^{-x} \\ du = e^x - e^{-x} dx \end{array} \right\} \int \frac{1}{u} du = \ln|e^x + e^{-x}| + C$$

$$102. \int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx \quad \left. \begin{array}{l} u = e^x + e^{-x} \\ du = e^x - e^{-x} dx \end{array} \right\} 2 \int \frac{1}{u} du = 2 \ln|e^x + e^{-x}| + C$$

$$104. \int \frac{e^{2x} + 2e^x + 1}{e^x} dx = \int (e^x + 2 + e^{-x}) dx = e^x + 2x - e^{-x} + C$$

$$106. \int e^{\sec 2x} \sec 2x \tan 2x dx \quad \left. \begin{array}{l} u = \sec(2x) \\ du = 2 \sec(2x) \tan(2x) dx \end{array} \right\} \frac{1}{2} \int e^u du = \frac{1}{2} e^{\sec(2x)} + C$$

$$108. \int \ln(e^{2x-1}) dx$$

$$\int (2x-1) dx = x^2 - x + C$$

$$30. \int \tan 5\theta \, d\theta = \int \frac{\sin(5\theta)}{\cos(5\theta)} \, d\theta = -\frac{1}{5} \int \ln|\cos(5\theta)| \, d\theta$$

$$32. \int \sec \frac{x}{2} \, dx$$

$$34. \int \frac{\csc^2 t}{\cot t} \, dt = -\ln|\cot(t)| + C$$

$$36. \int (\sec t + \tan t) \, dt$$

$$43. \int_0^2 \frac{5}{3x+1} \, dx \quad \left. \begin{array}{l} u=3x+1 \\ du=3dx \end{array} \right\} \frac{5}{3} \int \frac{1}{u} \, du = \frac{5}{3} \ln|3x+1| \Big|_0^2 = \frac{5}{3} \ln|3(2)+1| - \frac{5}{3} \ln|3(0)+1|$$

$$45. \int_1^e \frac{(1+\ln x)^2}{x} \, dx = \int_1^e u^2 \, du = \frac{1}{3} u^3 = \frac{1}{3} (1+\ln x)^3 \Big|_1^e = \frac{1}{3} (1+\ln e)^3 - \frac{1}{3} (1+\ln 1)^3$$

$$47. \int_0^2 \frac{x^2-2}{x+1} \, dx = \int_0^2 \frac{x-1}{x+1} \, dx$$

$$49. \int_1^2 \frac{1-\cos \theta}{\theta - \sin \theta} \, d\theta$$

$$44. \int_{-1}^1 \frac{1}{x+2} \, dx = \ln|x+2| \Big|_{-1}^1 = \ln|1+2| - \ln|-1+2|$$

$$46. \int_e^{e^2} \frac{1}{x \ln x} \, dx = \ln(\ln x) \Big|_e^{e^2} = \ln(\ln e^2) - \ln(\ln e)$$

$$48. \int_0^1 \frac{x-1}{x+1} \, dx$$

$$50. \int_{0.1}^{0.2} (\csc 2\theta - \cot 2\theta)^2 \, d\theta$$