Calculus Lesson 463: Applications of integrals (Other Uses)

Q20 A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000.

1. Find the demand function, assuming that it is linear.

2. How should the ticket prices be set to maximize revenue?

Q21 Given that $F(1) = 0.5$ and $f(x) = \sin(\ln(x))$, find $F(3)$

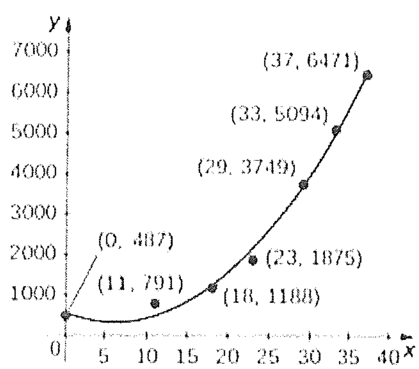
Q22 Given that $F(1) = 2$ and $f(x) = x^2$ then find the value of t where $F(t) - F(1) = 8$.

Q23 Given that the acceleration of a car is described by $a(t) = 4t - 12$ and that the current velocity of the car is 20 mph, find how long it will take the car to reach a velocity of 50 mph.

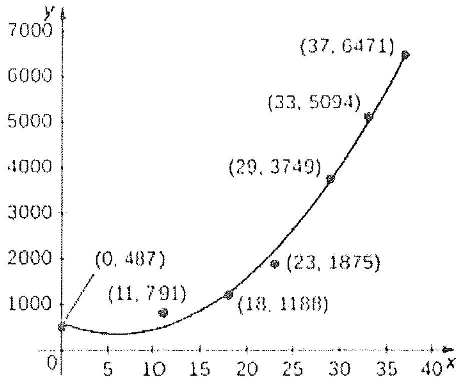
240. John is a 25-year old man who weighs 160 lb. He burns $500 - 50t$ calories/hr while riding his bike for t hours. If an oatmeal cookie has 55 cal and John eats $4t$ cookies during the tth hour, how many net calories has he lost after 3 hours riding his bike?

241. Sandra is a 25-year old woman who weighs 120 lb. She burns $300 - 50t$ cal/hr while walking on her treadmill. Her caloric intake from drinking Gatorade is $100t$ calories during the tth hour. What is her net decrease in calories after walking for 3 hours?

246. [T] The graph below plots the quadratic $p(t) = 6.48t^2 - 80.31t + 585.69$ against the data in preceding table, normalized so that $t = 0$ corresponds to 1963. Estimate the average number of bald eagles per year present for the 37 years by computing the average value of p over $[0, 37]$.



247. [T] The graph below plots the cubic $p(t) = 0.07t^3 + 2.42t^2 - 25.63t + 521.23$ against the data in the preceding table, normalized so that $t = 0$ corresponds to 1963. Estimate the average number of bald eagles per year present for the 37 years by computing the average value of p over $[0, 37]$.



238. Newton's law of gravity states that the gravitational force exerted by an object of mass M and one of mass m with centers that are separated by a distance r is $F = G\frac{mM}{r^2}$, with G an empirical constant $G = 6.67x10^{-11}\ m^3/(kg \cdot s^2)$. The work done by a variable force over an interval $[a, b]$ is defined as $W = \int_a^b F(x)dx$. If Earth has mass 5.97219×10^{24} and radius 6371 km, compute the amount of work to elevate a polar weather satellite of mass 1400 kg to its orbiting altitude of 850 km above Earth.

Interesting uses for the Fundamental Theorem of Calculus:

Q20 A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000.

1. Find the demand function, assuming that it is linear.

$$\left.\begin{matrix}(27{,}000,\ 10)\\(33{,}000,\ 8)\end{matrix}\right\}\ \text{slope} = \frac{-2}{6000} = \frac{-1}{3000}$$

$b = 19$

$$p(x) = -\frac{1}{3000}x + 19$$

2. How should the ticket prices be set to maximize revenue?

$$R(x) = x\left(\frac{-1}{3000}x + 19\right) = \frac{-1}{3000}x^2 + 19x$$

$$R'(x) = \frac{-1}{1500}x + 19$$

$$0 = \frac{-1}{1500}x + 19$$

$$x = 28500$$

$$p(28{,}500) = 9.50$$

$$R(28{,}500) = 270{,}750$$

Charge \$9.50 per ticket for attendance of 28,500 and revenue of \$270,750

Q21 Given that F(1) = 0.5 and f(x) = sin(ln(x)), find F(3)

$$\int_1^3 f(x)\,dx = F(3) - F(1)$$

$$1.153 = F(3) - .5$$

$$F(3) = 1.653$$

Q22 Given that F(1) = 2 and f(x) = x^2 then find the value of t where F(t) – F(1) = 8.

$$\int_1^t x^2\,dx = \frac{1}{3}x^3\Big|_1^t = \frac{1}{3}t^3 - \frac{1}{3}\cdot 1^3 = 8$$

$$\frac{1}{3}t^3 = 8\tfrac{1}{3}$$

$$t^3 = 25$$

$$t = 2.924$$

Q23 Given that the acceleration of a car is described by a(t) = 4t – 12 and that the current velocity of the car is 20 mph, find how long it will take the car to reach a velocity of 50 mph.

$$v(t) = 2t^2 - 12t + C$$

$$v(t) = 2t^2 - 12t + 20$$

need change in V to be 30

$$\int_0^x a(t)\,dt = v(x) - v(0) = 30$$

$$v(x) - 20 = 30$$

$$v(x) = 50$$

$$2t^2 - 12t + 20 = 50$$

$$2t^2 - 12t - 30 = 0$$

$$t = 7.898$$

*watch units in all

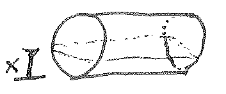
Level 5

232. A horizontal cylindrical tank has cross-sectional area $A(x) = 4(6x - x^2)m^2$ at height x meters above the bottom when $x \le 3$.

a. The volume V between heights a and b is $\int_a^b A(x)dx$. Find the volume at heights between 2 m and 3 m.

b. Suppose that oil is being pumped into the tank at a rate of 50 L/min. Using the chain rule, $\frac{dx}{dt} = \frac{dx}{dV}\frac{dV}{dt}$, at how many meters per minute is the height of oil in the tank changing, expressed in terms of x, when the height is at x meters?

c. How long does it take to fill the tank to 3 m starting from a fill level of 2 m?



a) $\int_2^3 (24x - 4x^2)dx$

$12x^2 - \frac{4}{3}x^3 \Big|_2^3 = 34\frac{2}{3}\ m^3$

b) convert L/min → m^3/min

$\frac{dV}{dt} = \frac{50L}{min}\left(\frac{1000mL}{1L}\right)\left(\frac{1cm^3}{1mL}\right)\left(\frac{1m}{100cm}\right)^3 = .05m^3/min$

$V = \left(12x^2 - \frac{4}{3}x^3\right)\Big|_a^x$

$\frac{dx}{dt} = \frac{dx}{dV}\cdot\frac{dV}{dt}$

$\frac{dV}{dx} = 24x - 4x^2$

$\boxed{\frac{dx}{dt} = \left(\frac{1}{24x-4x^2}\right)(.05)}$

c) $34\frac{2}{3} \div (.05) = 693.3$ min

Level 4

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$W = \int_{1000(6371)}^{(850+6371)1000} \frac{(6.67\times10^{-11})(5.97219\times10^{24})(1400)}{\cancel{6371000^2}\ r^2}dr$

units: $\frac{m^3\cdot Kg\cdot Kg\cdot Kg}{Kg s^2\ m^2} = \frac{m Kg^2}{s^2}$

$W = -\frac{1}{3}(5.5768\times10^{17})r^{-3}\Big|_{6371000}^{7221000}$

$W = 1.03\times10^{10}$ Joules

level 3

239. For a given motor vehicle, the maximum achievable deceleration from braking is approximately 7 m/sec² on dry concrete. On wet asphalt, it is approximately 2.5 m/sec². Given that 1 mph corresponds to 0.447 m/sec, find the total distance that a car travels in meters on dry concrete after the brakes are applied until it comes to a complete stop if the initial velocity is 67 mph (30 m/sec) or if the initial braking velocity is 56 mph (25 m/sec). Find the corresponding distances if the surface is slippery wet asphalt.

~~Dry $a(t) = -7$~~

~~$v(t) = .447(67) - 7t$~~ ~~$56 - 7t$~~

~~$s(t) = 0$~~

~~$0 = .447 - 7t$~~

~~$9.524 sec = t$~~

$\int_0^{4.286} v(t)dt = 64m$

$\int_0^{3.571} v(t)dt = 44m$

Dry $a(t) = -7$

$v(t) = (30) - 7t$ $\qquad v(t) = (25) - 7t$

$0 = 30 - 7t$ $\qquad 0 = 25 - 7t$

$t = 4.286$ sec. $\qquad t = 3.571$

Wet $a(t) = -2.5$

$v(t) = 30 - 2.5t$ $\qquad v(t) = 25 - 2.5t$

$0 = 30 - 2.5t$ $\qquad 0 = 25 - 2.5t$

$t = 12$ sec $\quad D = 180$ $\qquad t = 10.$ sec $\quad D = 125$

Level 3

240. John is a 25-year old man who weighs 160 lb. He burns $500 - 50t$ calories/hr while riding his bike for t hours. If an oatmeal cookie has 55 cal and John eats $4t$ cookies during the tth hour, how many net calories has he lost after 3 hours riding his bike?

Burn

$\frac{dc}{dt} = 500 - 50t$

$\int_0^3 (500 - 50t)dt$

1275 cal. burned

Eat

$\frac{dc}{dt} = 55\cdot 4t$

$\int_0^3 220t\,dt$

990 cal eaten

$990 - 1275 = -285$ cal.

Level 3

241. Sandra is a 25-year old woman who weighs 120 lb. She burns $300 - 50t$ cal/hr while walking on her treadmill. Her caloric intake from drinking Gatorade is $100t$ calories during the tth hour. What is her net decrease in calories after walking for 3 hours?

$$\int_0^3 (100t - (300-50t))\,dt = -225$$ Calories

Level 3

243. Although some engines are more efficient at given a horsepower than others, on average, fuel efficiency decreases with horsepower at a rate of 1/25 mpg/horsepower. If a typical 50-horsepower engine has an average fuel efficiency of 32 mpg, what is the average fuel efficiency of an engine with the following horsepower: 150, 300, 450?

$$\frac{de}{dh} = \frac{-1}{25}$$

@ $32 = \frac{-1}{25}(50) + V_0$

$34 = V_0$

$e = \frac{-1}{25}h + \cancel{32}V_0$

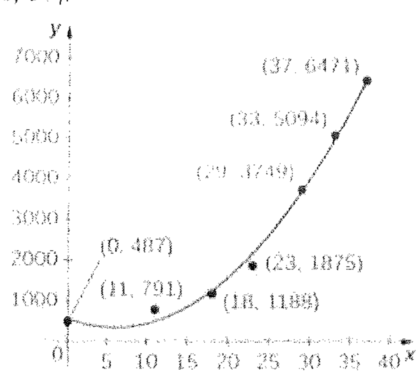
$e = \frac{-1}{25}(150) + 34 = 28$ mpg

$(300) = 22$ mpg

$(450) = 16$ mpg

Level 3

246. [T] The graph below plots the quadratic $p(t) = 6.48t^2 - 80.31t + 585.69$ against the data in preceding table, normalized so that $t = 0$ corresponds to 1963. Estimate the average number of bald eagles per year present for the 37 years by computing the average value of p over $[0, 37]$.



$$\frac{1}{37}\int_0^{37} p(t)\,dt = \frac{1}{37}(76108.815)$$

$= 2057$ eagles

Level 4

242. A motor vehicle has a maximum efficiency of 33 mpg at a cruising speed of 40 mph. The efficiency drops at a rate of 0.1 mpg/mph between 40 mph and 50 mph, and at a rate of 0.4 mpg/mph between 50 mph and 80 mph. What is the efficiency in miles per gallon if the car is cruising at 50 mph? What is the efficiency in miles per gallon if the car is cruising at 80 mph? If gasoline costs \$3.50/gal, what is the cost of fuel to drive 50 mi at 40 mph, at 50 mph, and at 80 mph?

$33 = -.1(40) + V_0$

$\frac{de}{ds} = -.1$ on $40 \le s \le 50$

$e = -.1(s) + V_0$ $\quad e = -.1s + 37$

$\frac{de}{ds} = -.4$ on $50 < s \le 80$

$e = -.4s + V_0$

at 50mph

$e = -.1(50) + 37$

$\boxed{e = 32}$ mpg

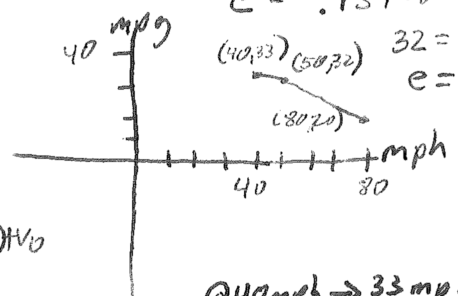
$32 = -.4(50) + V_0$

$e = 52 - .4s$

at 80mph

$e = 52 - .4(80)$

$\boxed{e = 20}$ mpg



@40mph → 33 mpg

50mi = 1.51 gal

$\frac{d\$}{dg} = 3.5$

$3.5(1.51) = \boxed{\$5.30}$

@50mph → 32mpg → 1.5625 gal

$\times 3.5 = \boxed{\$5.47}$

@80 mph → 20mpg → 2.5gal = $\boxed{\$8.75}$

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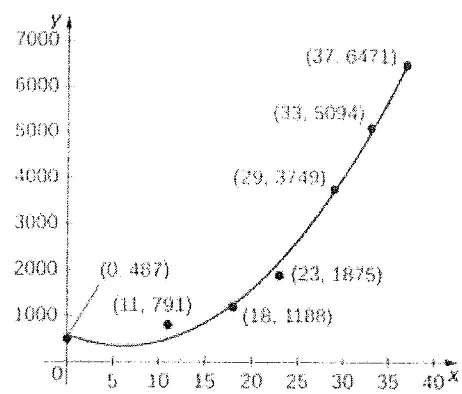

$= \frac{1}{37}(75399.679)$

$= 2038$ eagles