Recommended Review

State the 3 possible answers for limits as x approaches infinity and how to tell which will occur.

If $f(x) = ax^n$, then define f'(x) and F(x)

State the first derivative test and draw a picture:

$$\lim_{x\to 0}\frac{(2+x)^4-16}{x}$$

Find d/dx for $4x + xy = 5$

$$\lim_{x\to \pi}\frac{\cos(x)-\cos(\pi)}{x-\pi}$$

Find d/dt for $4x + xy = 5$

t (hours)	0	2	5	7	8	10
$v(t)$ (miles per hour)	50	55	60	70	65	75

C

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road. 2.2

(a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure. 4.3

(b) Use a right Riemann sum with the subintervals given in the table to approximate $\int_0^{10} v(t)\,dt$. Indicate units of measure. What physical quantity does this integral represent? 3.4

(c) The function $v(t)$ is twice differentiable on the interval $[0, 10]$. Show that there must be a moment of time when the acceleration of the car is equal to zero.

4.4

4. $\int_1^3 \frac{8}{x^3}\,dx =$

(A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$

5. $\int_1^4 \frac{dx}{\sqrt{x}} =$

4.4

(A) $\frac{1}{2}$

(B) 2

(C) 4

(D) $\frac{14}{3}$

(E) $\frac{21}{2}$

15. $\int_{e^{-1}}^{1} \frac{x^2 - x}{x^2}\,dx =$

(A) $-\frac{1}{e}$

4.4

(B) $\frac{1}{e}$

(C) $2 - \frac{1}{e}$

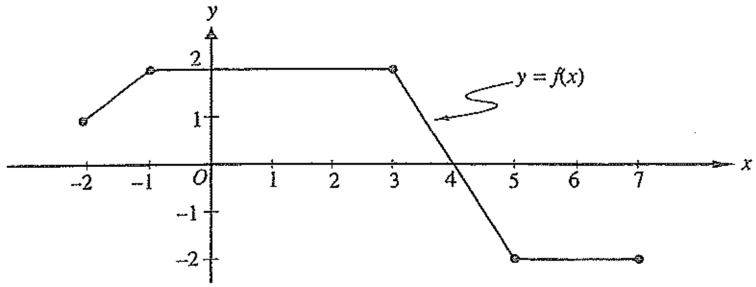
(D) e

(E) $e^2 - \frac{1}{e}$

4.4

C 9. $\int_2^{e+1}\left(\frac{4}{x-1}\right)dx =$

(A) 4 (B) $4e$ (C) 0 (D) $-4e$ (E) -4



* Requires Notes from Day 4

21. The graph of a piecewise-linear function $f(x)$, for $-2 \le x \le 7$, is shown in the figure above. If $F(x) = \int_2^x f(t)\,dt$, which of the following statements is true?

(A) $F(2) > F(3) > F(7)$

(B) $F(2) > F(7) > F(3)$

(C) $F(3) > F(2) > F(7)$

(D) $F(3) > F(7) > F(2)$ 4.4

(E) $F(7) > F(2) > F(3)$

C

79. What is the average value of the function f defined by $f(x) = \sin(x^2)$ on the closed interval $[1, 3]$?

(A) 0.154

(B) 0.232 ~~3.12~~

(C) 0.463 4.4

(D) 0.696

(E) 1.392

4.3

x	0	1	2	3
$f(x)$	2	5	4	3

C

91. The function f is continuous on the closed interval $[0, 3]$ and has values that are given in the table above. Using the subintervals $[0, 1]$, $[1, 2]$, and $[2, 3]$, what is the trapezoidal approximation to $\int_0^3 f(x)\,dx$?

(A) 11

(B) 11.5

(C) 12

(D) 12.5

(E) 13

x	0	1	3	7	10
$f(x)$	1	-1	4	2	3

4.3

19. The function f is continuous on the closed interval $[0, 10]$ and has values given in the table above. Using the subintervals $[0, 1]$, $[1, 3]$, $[3, 7]$, and $[7, 10]$, what is the left Riemann sum estimate for $\int_0^{10} f(x)\,dx$?

(A) 15

(B) 17.5

(C) 20

(D) 21

(E) 22.5

C

11. The area of the region in the first quadrant bounded by the graph of $y = x\sqrt{9+x^2}$, the x-axis, and the line $x = 4$ is

(A) $9\sec^3 4 - 9$

(B) $\frac{16}{3}$

(C) $\frac{13}{3}\sqrt{13} - 9$

(D) $\frac{64}{3}$

(E) $\frac{98}{3}$

4.4

53. State the Fundamental Theorem of Calculus.

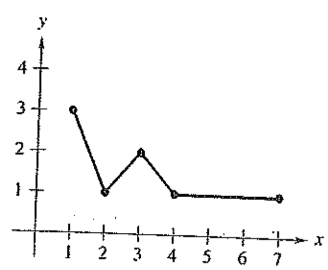


Figure for 54

54. The graph of f is given in the figure.

(a) Evaluate $\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

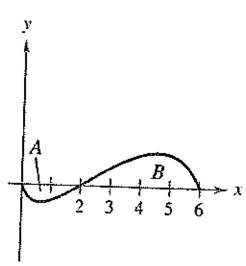


Figure for 55–60

In Exercises 55–60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.

59. $\int_0^6 [2 + f(x)]\,dx =$

60. The average value of f over the interval $[0, 6]$ is .

Recommended Review

State the 3 possible answers for limits as x approaches infinity and how to tell which will occur.

1   0   bottom wins $\frac{1}{x}$

2   constant, tie $\frac{3x^2}{4x^2-1} = \frac{3}{4}$

3   ∞ or $-\infty$, top wins $\frac{3x^6-1}{x} \longrightarrow \infty$ to the right

$-\infty$ to ~~and~~ the left

If $f(x) = ax^n$, then define $f'(x)$ and $F(x)$

$f'(x) = an \cdot x^{n-1}$ $F(x) = \frac{a}{n+1} x^{n+1}$

State the first derivative test and draw a picture:

$$\lim_{x \to 0} \frac{(2+x)^4 - 16}{x}$$

$f(x) = x^4$

$f'(x) = 4x^3$

$x = 2$

$f'(2) = 4 \cdot 2^3 = 32$

Find d/dx for $4x + xy = 5$

$4 + y + xy' = 0$

$y' = \frac{-4-y}{x}$

$$\lim_{x \to \pi} \frac{\cos(x) - \cos(\pi)}{x - \pi}$$

$f(x) = \cos x$

$f'(x) = -\sin x$

$x = \pi$

$f'(\pi) = -\sin(\pi) = 0$

Find d/dt for $4x + xy = 5$

$4x' + x'y + xy' = 0$

$4\frac{dx}{dt} + \frac{dx}{dt} y + x\frac{dy}{dt} = 0$

t (hours)	0	2	5	7	8	10
$v(t)$ (miles per hour)	50	55	60	70	65	75

C

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road. 2.2

(a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure. 4.3

$\frac{70-60}{7-5} = \frac{10}{2} = 5$ mph/h

(b) Use a right Riemann sum with the subintervals given in the table to approximate $\int_0^{10} v(t)\,dt$. Indicate units of measure. What physical quantity does this integral represent? 3.4

B	2	3	2	1	2
H	55	60	70	65	75
Area	110	180	140	65	150

total: 645

(c) The function $v(t)$ is twice differentiable on the interval $[0, 10]$. Show that there must be a moment of time when the acceleration of the car is equal to zero.

accel. from $t=5$ to $t=7$ is 5mph/h accel. from $t=7$ to $t=8$ is -5mph/h. $a(t)$ is diff, so it is cont. so $a(t)=0$ for $5<t<8$ by IVT.

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4. $\int_1^3 \frac{8}{x^3}\,dx =$ 4.4

$8x^{-3}$

$\left[\frac{8}{-2}x^{-2}\right]_1^3 = \left[-4/x^2\right]_1^3 = \frac{-4}{9} - \frac{-4}{1}$

$-\frac{4}{9} + \frac{36}{9} = \frac{32}{9}$

(A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$

5. $\int_1^4 \frac{dx}{\sqrt{x}} =$ 4.4

$x^{-1/2}$

$\left[2x^{1/2}\right]_1^4 = 2(4)^{1/2} - 2(1)^{1/2} = 4 - 2 = 2$

(A) $\frac{1}{2}$

(B) 2

(C) 4

(D) $\frac{14}{3}$

(E) $\frac{21}{2}$

15. $\int_{e^{-1}}^{1} \frac{x^2 - x}{x^2}\,dx =$ 4.4

$1 - x^{-1}$

$\left[x - \ln x\right]_{e^{-1}}^{1} = \left(1 - \ln 1\right) - \left[e^{-1} - \ln(e^{-1})\right]$

$[1 - 0] - e^{-1} - (-1)$

$1 - \frac{1}{e} + 1$

(A) $-\frac{1}{e}$

(B) $\frac{1}{e}$

(C) $2 - \frac{1}{e}$

(D) e

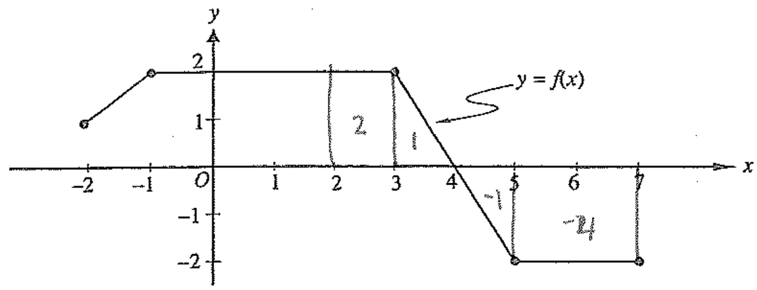
(E) $e^2 - \frac{1}{e}$

4.4

C

9. $\int_{2}^{e+1} \left(\frac{4}{x-1}\right) dx =$

(A) 4 (B) $4e$ (C) 0 (D) $-4e$ (E) -4



21. The graph of a piecewise-linear function $f(x)$, for $-2 \le x \le 7$, is shown in the figure above. If $F(x) = \int_{2}^{x} f(t)\,dt$, which of the following statements is true?

Area from 2 to x

(A) $F(2) > F(3) > F(7)$

(B) $F(2) > F(7) > F(3)$

(C) $F(3) > F(2) > F(7)$

(D) $F(3) > F(7) > F(2)$

(E) $F(7) > F(2) > F(3)$

$F(2) = \int_2^2 f(t)dt = 0$

$F(3) = \int_2^3 f(t)dt = 2$

$F(7) = \int_2^7 f(t)dt = 3 + -5 = -2$

4.4

C

79. What is the average value of the function f defined by $f(x) = \sin(x^2)$ on the closed interval $[1, 3]$?

(A) 0.154

(B) 0.232

(C) 0.463

(D) 0.696

(E) 1.392

$\frac{1}{3-1} \int_1^3 \sin(x^2)dx$

$\frac{1}{2}[.463) = .232$

4.4

x	0	1	2	3
$f(x)$	2	5	4	3

4.3

C

91. The function f is continuous on the closed interval $[0, 3]$ and has values that are given in the table above. Using the subintervals $[0, 1]$, $[1, 2]$, and $[2, 3]$, what is the trapezoidal approximation to $\int_0^3 f(x)\,dx$?

(A) 11

(B) 11.5

(C) 12

(D) 12.5

(E) 13

	ave H	w	Area
0 to 1	3.5	1	3.5
1 to 2	4.5	1	4.5
2 to 3	3.5	1	3.5
			11.5

x	0	1	3	7	10
$f(x)$	1	-1	4	2	3

4.3

19. The function f is continuous on the closed interval $[0, 10]$ and has values given in the table above. Using the subintervals $[0, 1]$, $[1, 3]$, $[3, 7]$, and $[7, 10]$, what is the left Riemann sum estimate for $\int_0^{10} f(x)\,dx$?

(A) 15

(B) 17.5

(C) 20

(D) 21

(E) 22.5

	L Hght	W	Area
0 to 1	1	1	1
1 to 3	-1	2	-2
3 to 7	4	4	16
7 to 10	2	3	6

Area: 21

C

11. The area of the region in the first quadrant bounded by the graph of $y = x\sqrt{9 + x^2}$, the x-axis, and the line $x = 4$ is

(A) $9 \sec^3 4 - 9$

(B) $\frac{16}{3}$

(C) $\frac{13}{3}\sqrt{13} - 9$

(D) $\frac{64}{3}$

(E) $\frac{98}{3}$

$\int$

$32.\overline{6}$

4.4

53. State the Fundamental Theorem of Calculus.

a) $2+1.5+1.5+3 = 8$

b) $\frac{1}{7-1}\int_1^7 f(x)dx = \frac{1}{6}\cdot 8 = \frac{8}{6}$

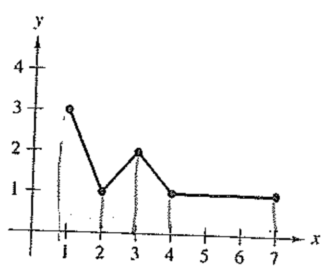


Figure for 54

54. The graph of f is given in the figure.

(a) Evaluate $\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

c) 2 | 12 | 1 | 7

$8+12 = 20$ area

$\frac{20}{6}$ = ave. val.

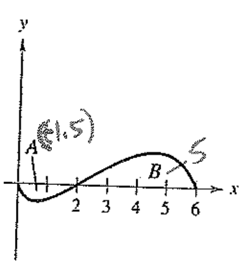


Figure for 55–60

In Exercises 55–60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.

59. $\int_0^6 [2 + f(x)]\,dx =$ $\int_0^6 2dx + \int_0^6 f(x)dx = 12 + 3.5 = 15.5$

60. The average value of f over the interval $[0, 6]$ is $\frac{1}{6-0}\int_0^6 f(x)dx = \frac{1}{6}(3.5) = .58\overline{3}$