Calculus Lesson 441: Fundamental Theorem of Calculus, part 1

If the function f is continuous on the closed interval [a, b] and F is an antiderivative of f on the interval [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Example - f(x) = x Find the area under the curve from x = 2 to x = 3.

$$\int_a^b f(x)dx \;=\; \int_2^3 xdx \;=\; \frac{1}{2}x^2\Big]_2^3 \;=\; \frac{1}{2}3^2 - \frac{1}{2}2^2 \;=\; \frac{9}{2} - \frac{4}{2} \;=\; 2.5$$

Reminder - Finding the Area Under the Curve on the Calculator

1. Type f(x) into Y1.
2. Graph the function so that the interval of integration appears on the screen (x-min, x-max).
3. Press 2nd   CALC   and find the Integral option (#7) and press ENTER.
4. Type in the x value of the left bound and press ENTER.
5. Type in the x value of the right bound and press ENTER.

Examples

Find area under .2x from x = 2 to 5

$$\int_1^5 (x^4 - 1)dx$$

$$\int_1^4 \left(\frac{3}{x^2} + \sqrt{x}\right) dx$$

$$\int_1^5 |x|dx$$

Problems

170. $\int_{-1}^{2}\left(x^2 - 3x\right)dx$

171. $\int_{-2}^{3}\left(x^2 + 3x - 5\right)dx$

172. $\int_{-2}^{3}(t + 2)(t - 3)dt$

173. $\int_{2}^{3}\left(t^2 - 9\right)\left(4 - t^2\right)dt$

174. $\int_{1}^{2} x^9\, dx$

175. $\int_{0}^{1} x^{99}\, dx$

176. $\int_{4}^{8}\left(4t^{5/2} - 3t^{3/2}\right)dt$

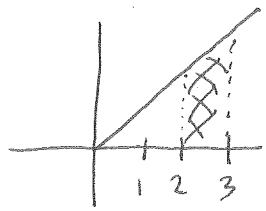
177. $\int_{1/4}^{4}\left(x^2 - \frac{1}{x^2}\right)dx$

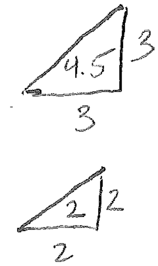
178. $\int_{1}^{2}\frac{2}{x^3}dx$

Calculus Lesson 441: Fundamental Theorem of Calculus, part 1

If the function f is continuous on the closed interval [a, b] and F is an antiderivative of f on the interval [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a)$$





$\frac{1}{2}x^2 \Big|_2^3$

Example - f(x) = x Find the area under the curve from x = 2 to x = 3.

$$\int_a^b f(x)dx = \int_2^3 xdx = \frac{1}{2}x^2\Big]_2^3 = \frac{1}{2}3^2 - \frac{1}{2}2^2 = \frac{9}{2} - \frac{4}{2} = 2.5$$

Reminder - Finding the Area Under the Curve on the Calculator

1. Type f(x) into Y1.
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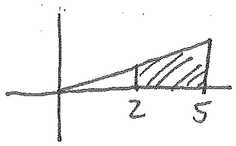
Examples



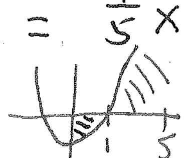
$\int_2^5 .2xdx = .1x^2\Big|_2^5 = .1(5)^2 - .1(2)^2$

$2.5 - .4 = 2.1$

Find area under .2x from x = 2 to 5

$$\int_1^5 (x^4 - 1)dx = \frac{1}{5}x^5 - x\Big|_1^5 = \frac{1}{5}(5)^5 - 5 - \left(\frac{1}{5}(1)^5 - 1\right)$$

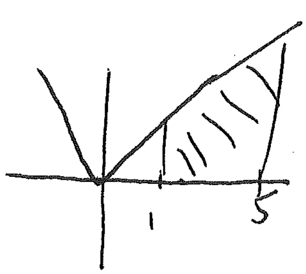
$620 - -\frac{4}{5} = 620.8$



$$\int_1^4 \left(\frac{3}{x^2} + \sqrt{x}\right) dx = \int_1^4 (3x^{-2} + x^{1/2})dx = -3x^{-1} + \frac{2}{3}x^{3/2}\Big|_1^4 = 4.58\overline{3} - -2.\overline{3}$$

$= 6.91\overline{6}$

$$\int_1^5 |x|dx$$



$\int_1^5 xdx$

Problems

170. $\int_{-1}^{2}(x^2-3x)dx = \frac{1}{3}x^3-\frac{3}{2}x^2\Big|_{-1}^{2} = \left(\frac{1}{3}(2)^3-\frac{3}{2}(2)^2\right)-\left(\frac{1}{3}(-1)^3-\frac{3}{2}(-1)^2\right)$

$\frac{8}{3}-6+\frac{1}{3}+\frac{3}{2}=\boxed{-1.5}$

171. $\int_{-2}^{3}(x^2+3x-5)dx = \frac{1}{3}x^3+\frac{3}{2}x^2-5x\Big|_{-2}^{3} = \left(\frac{1}{3}(3)^3+\frac{3}{2}(3)^2-5(3)\right)-\left(\frac{1}{3}(-2)^3+\frac{3}{2}(-2)^2-5(-2)\right)$

$7.5-13.\overline{3}=\boxed{-5.8\overline{3}}$

172. $\int_{-2}^{3}(t+2)(t-3)dt = \int_{-2}^{3}(t^2-t-6)dt = \frac{1}{3}t^3-\frac{1}{2}t^2-6t\Big|_{-2}^{3} =$

$\left(\frac{1}{3}(3)^3-\frac{1}{2}(3)^2-6(3)\right)-\left(\frac{1}{3}(-2)^3-\frac{1}{2}(-2)^2-6(-2)\right)=$

$-13.5-7.\overline{3}=\boxed{-20.8\overline{3}}$

173. $\int_{2}^{3}(t^2-9)(4-t^2)dt = \int_{2}^{3}(13t^2-t^4-36) = \frac{13}{3}t^3-\frac{1}{5}t^5-36t\Big|_{2}^{3}$

$=\left(\frac{13}{3}(3)^3-\frac{1}{5}(3)^5-36(3)\right)-\left(\frac{13}{3}(2)^3-\frac{1}{5}(2)^5-36(2)\right)$

$=\boxed{4.1\overline{3}}$

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174. $\int_{1}^{2}x^9\,dx = \frac{1}{10}x^{10}\Big|_{1}^{2} = \frac{1}{10}(2)^{10}-\frac{1}{10}(1)^{10} = 102.4-.1=\boxed{102.3}$

175. $\int_{0}^{1}x^{99}\,dx = \frac{1}{100}x^{100}\Big|_{0}^{1} = \frac{1}{100}1^{100}-\frac{1}{100}0^{100}=\boxed{\frac{1}{100}}$

176. $\int_{4}^{8}(4t^{5/2}-3t^{3/2})dt = \frac{4}{1}\cdot\frac{2}{7}t^{\frac{7}{2}}-\frac{3}{1}\cdot\frac{2}{5}t^{5/2}\Big|_{4}^{8} = \left(\frac{8}{7}\cdot 8^{\frac{7}{2}}-\frac{6}{5}\cdot 8^{\frac{5}{2}}\right)-\left(\frac{8}{7}\cdot 4^{\frac{7}{2}}-\frac{6}{5}\cdot 4^{\frac{5}{2}}\right)$

$1437.81-107.89=\boxed{1329.92}$

177. $\int_{1/4}^{4}\left(x^2-\frac{1}{x^2}\right)dx = \frac{1}{3}x^3+x^{-1}\Big|_{1/4}^{4} = \left(\frac{1}{3}\cdot 4^3+\frac{1}{4}\right)-\left(\frac{1}{3}\left(\frac{1}{4}\right)^3+4\right)=\boxed{17.578}$

x^2-x^{-2}

178. $\int_{1}^{2}\frac{2}{x^3}dx = -x^{-2}\Big|_{1}^{2} = -\frac{1}{2^2}--\frac{1}{1^2}$

$2x^{-3}$

$-\frac{1}{4}+1=\boxed{\frac{3}{4}}$

Calculus Worksheet 4-4A Name______________________________ Pd______

Graphical Reasoning **In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.**

1. $\displaystyle\int_0^{\pi} \frac{4}{x^2+1}\,dx$

2. $\displaystyle\int_0^{\pi} \cos x\,dx$

3. $\displaystyle\int_{-2}^{2} x\sqrt{x^2+1}\,dx$

4. $\displaystyle\int_{-2}^{2} x\sqrt{2-x}\,dx$

In Exercises 5–26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\displaystyle\int_0^{1} 2x\,dx$

6. $\displaystyle\int_2^{7} 3\,dv$

8. $\displaystyle\int_2^{5} (-3v+4)\,dv$

7. $\displaystyle\int_{-1}^{0} (x-2)\,dx$

9. $\displaystyle\int_{-1}^{1} (t^2-2)\,dt$

11. $\displaystyle\int_0^{1} (2t-1)^2\,dt$

13. $\displaystyle\int_1^{2} \left(\frac{3}{x^2}-1\right)dx$

15. $\displaystyle\int_1^{4} \frac{u-2}{\sqrt{u}}\,du$

17. $\displaystyle\int_{-1}^{1} \left(\sqrt[3]{t}-2\right)dt$

C 19. $\displaystyle\int_0^{1} \frac{x-\sqrt{x}}{3}\,dx$

C 21. $\displaystyle\int_{-1}^{0} \left(t^{1/3}-t^{2/3}\right)dt$

C 18. $\displaystyle\int_1^{8} \sqrt{\frac{2}{x}}\,dx$

C 20. $\displaystyle\int_0^{2} (2-t)\sqrt{t}\,dt$

C 22. $\displaystyle\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$

Calculus Worksheet 4-4A Name Key Pd

Graphical Reasoning In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.

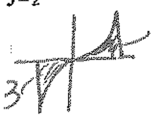
1. $\int_0^{\pi} \frac{4}{x^2+1}\,dx$

5.050

2. $\int_0^{\pi} \cos x\,dx$

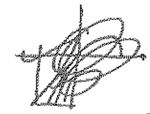
total: 0

3. $\int_{-2}^{2} x\sqrt{x^2+1}\,dx$



-3.393 3.393 total: 0

4. $\int_{-2}^{2} x\sqrt{2-x}\,dx$



-3.641 1.508 total: -2.133

In Exercises 5–26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\int_0^1 2x\,dx$ $\left[\frac{2}{2}x^2\right]_0^1 = 1^2 - 0^2 = 1$

6. $\int_2^7 3\,dv$ $\left[3v\right]_2^7 = 21 - 6 = 15$

8. $\int_2^5 (-3v+4)\,dv$ $\left[-\frac{3}{2}v^2+4v\right]_2^5 = -\frac{3}{2}(25)+4(5) - \left(-\frac{3}{2}(4)+4(2)\right) = -37.5+20-(-6+8) = -19.5$

7. $\int_{-1}^{0} (x-2)\,dx$ $\left[\frac{1}{2}x^2-2x\right]_{-1}^{0} = 0 - \left[\frac{1}{2}(-1)^2-2(-1)\right] = 0 - \left(\frac{1}{2}+2\right) = -2.5$

9. $\int_{-1}^{1} (t^2-2)\,dt$ $\left[\frac{1}{3}t^3-2t\right]_{-1}^{1} = \left[\frac{1}{3}\cdot1^3-2\cdot1\right] - \left[\frac{1}{3}(-1)^3-2(-1)\right] = -1.\overline{6} - 1.\overline{6} = -3.\overline{3}$

11. $\int_0^1 (2t-1)^2\,dt$ $4t^2-4t+1$ $\left[\frac{4}{3}t^3-\frac{4}{2}t^2+t\right]_0^1 = \left[\frac{4}{3}(1)^3-2(1)^2+1\right]-[0] = \frac{1}{3}$

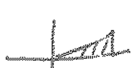
13. $\int_1^2 \left(\frac{3}{x^2}-1\right)dx$ $3x^{-2}-1$ $= \left[\frac{3}{-1}x^{-1}-x\right]_1^2 = \left[-3\left(\frac{1}{2}\right)-2\right]-\left[-3\left(\frac{1}{1}\right)-(1)\right] = -3.5+4 = \frac{1}{2}$

$\frac{u}{u^{1/2}} - \frac{2}{u^{1/2}}$

15. $\int_1^4 \frac{u-2}{\sqrt{u}}\,du$ $u^{1/2}-2u^{-1/2}$ $= \left[\frac{2}{3}u^{\frac{3}{2}} - 2\left(\frac{2}{1}\right)u^{1/2}\right]_1^4 = \left[\frac{2}{3}(4)^{3/2}-4(4)^{1/2}\right]-\left[\frac{2}{3}(1)^{3/2}-4(1)^{1/2}\right] = \left[\frac{16}{3}-8\right]-\left[\frac{2}{3}-4\right] = 2/3$

17. $\int_{-1}^{1} (\sqrt[3]{t}-2)\,dt$ $t^{1/3}-2$ $= \left[\frac{3}{4}t^{4/3}-2t\right]_{-1}^{1} = \left[\frac{3}{4}(1)^{4/3}-2(1)\right]-\left[\frac{3}{4}(-1)^{4/3}-2(-1)\right] = \frac{-5}{4}-\frac{11}{4} = -4$

C 19. $\int_0^1 \frac{x-\sqrt{x}}{3}\,dx$



Area $= -.0556$

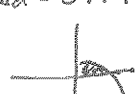
C 21. $\int_{-1}^{0} (t^{1/3}-t^{2/3})\,dt$

Area $= -1.350$

C 18. $\int_1^8 \sqrt{\frac{2}{x}}\,dx$

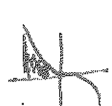
Area = 5.171

C 20. $\int_0^2 (2-t)\sqrt{t}\,dt$



Area = 1.508

C 22. $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$



Area = 57.112