Calculus Lesson 424: Approximating the Area under a curve Using a Calculator and from tables

Putting it all together

Finding the width of the rectangles: $width = \frac{Right - Left}{n}$

Connecting Sigma Notation to the area approximation methods

The total approximate area is found by adding together the area of each rectangle. This can be written in sigma notation:

$$area = \sum width \bullet height$$

$$area\ on\ interval\ x = a\ to\ x = b\ using\ n\ subintervals: \sum_{i=1}^{n} \frac{b-a}{n} \bullet f(x_i)$$

How this relates to calculus

The more rectangles that are used, the more accurate the approximation.

The approximation becomes exact when n approaches infinity.

So the exact area is $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{b-a}{n} \bullet f(x_i)$

As n approaches infinity, the width approaches zero.

As n approaches infinity, the error approaches zero.

<u>*Note on this theorem:*</u> f(x) must be continuous and nonnegative on [a, b]

Some Books give the Riemann Sum Formula as

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

Exact Area on a calculator

1. Create an appropriate graph on the calculator.
2. Select "Sf(x)dx" from the Calculate Menu
3. Enter the Left Bound and press Enter
4. Enter the Right Bound and press Enter
5. The calculator will now give the exact area.

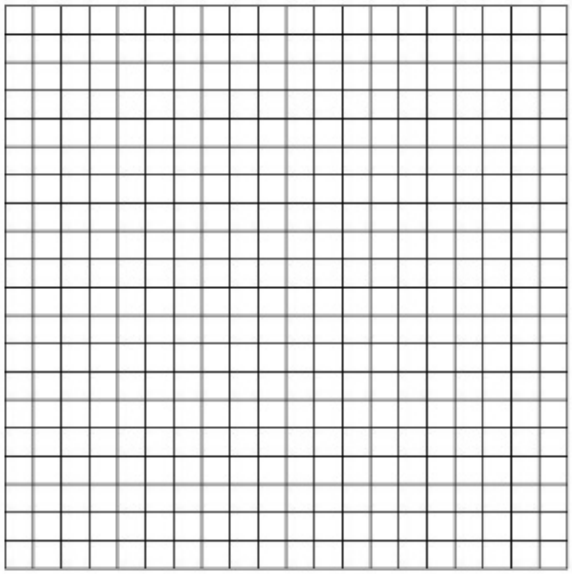
Area from Table Hints

1. Draw a picture of the values, then make the rectangles/trapezoids on the picture.
2. If using the Left rule, you will not use the x furthest to the right (usually listed last).
3. If using the Right rule, you will not use the x furthest to the left (usually list first).
4. Remember: you can find the trapezoid rule by averaging the Left rule and Right rule answers.

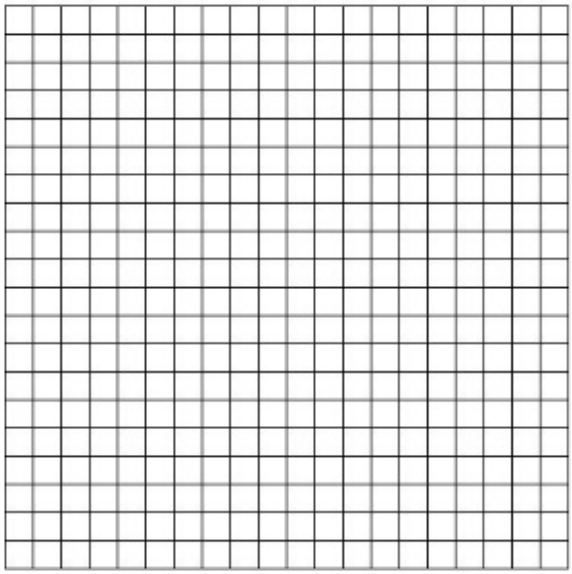
Table Practice

x	0	1	2	3	4
f(x)	3	2	6	4	7

1. Sketch the table values and find the area under the curve by the left hand rule (4 rectangles):



2. Sketch the table values and find the area under the curve by the right hand rule (4 rectangles):



x	f(x)
1	3
1.5	4
2	4
2.5	6
3	2
3.5	3
4	5

3. Find the left hand approximation for the area under the curve described in the table on [1, 4] by using 6 equal subintervals.

4. Find the right hand approximation for the area under the curve described in the table on [1, 4] by using 6 equal subintervals.

5. Find the trapezoidal approximation for the area under the curve described in the table on [1, 4] by using 6 equal subintervals.

NOTE: there is a shortcut for this step.

6. Find the midpoint approximation for the area under the curve described in the table on [1, 4] by using 3 equal subintervals.

Exact Area on a calculator

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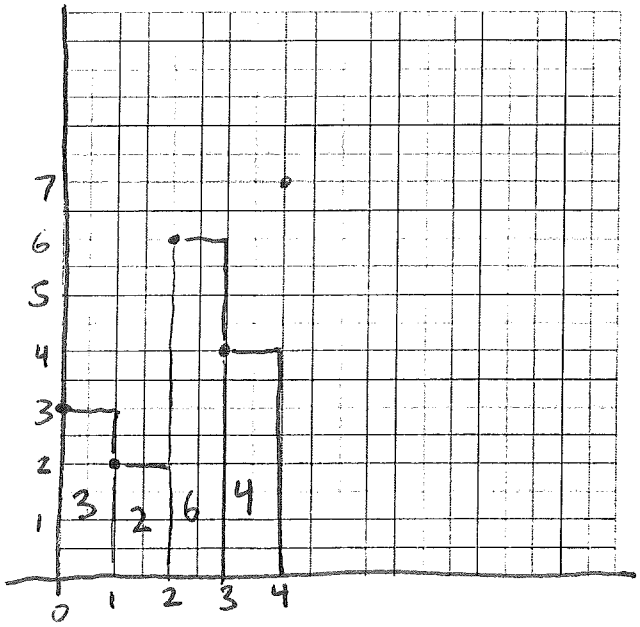
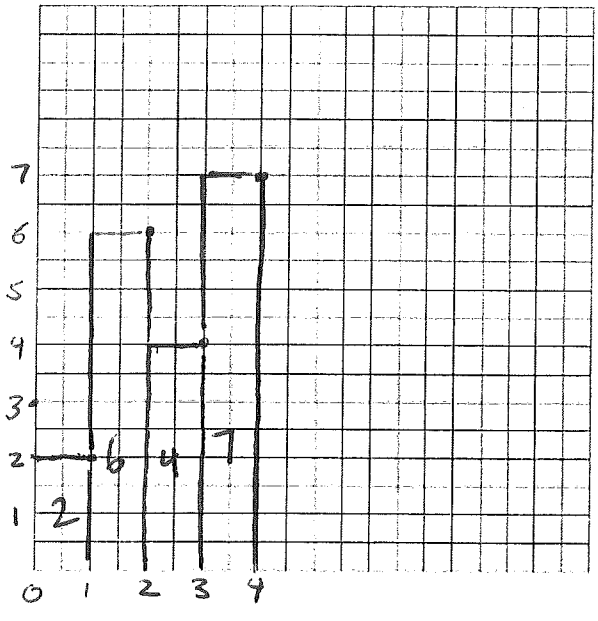


15

2. Sketch the table values and find the area under the curve by the right hand rule (4 rectangles):



19

x	f(x)
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1.5	4
2	4
2.5	6
3	2
3.5	3
4	5

3. Find the left hand approximation for the area under the curve described in the table on [1, 4] by using 6 equal subintervals.

ignore (4, 5)

(22) · $\frac{1}{2}$ = (11)

4. Find the right hand approximation for the area under the curve described in the table on [1, 4] by using 6 equal subintervals.

ignore (1, 3)

(24) · $\frac{1}{2}$ = (12)

5. Find the trapezoidal approximation for the area under the curve described in the table on [1, 4] by using 6 equal subintervals.

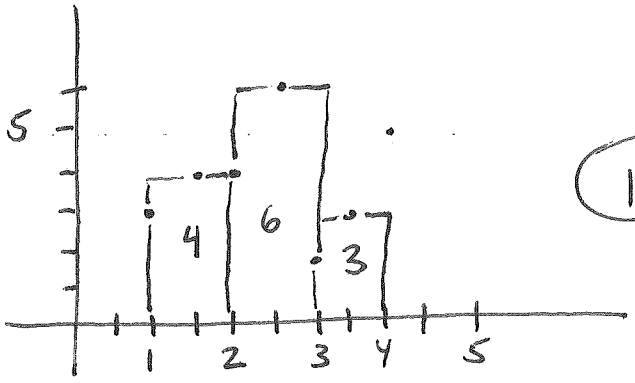
Average
11 + 12

11.5

NOTE: there is a shortcut for this step.

6. Find the midpoint approximation for the area under the curve described in the table on [1, 4] by using 3 equal subintervals.

width = 1



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