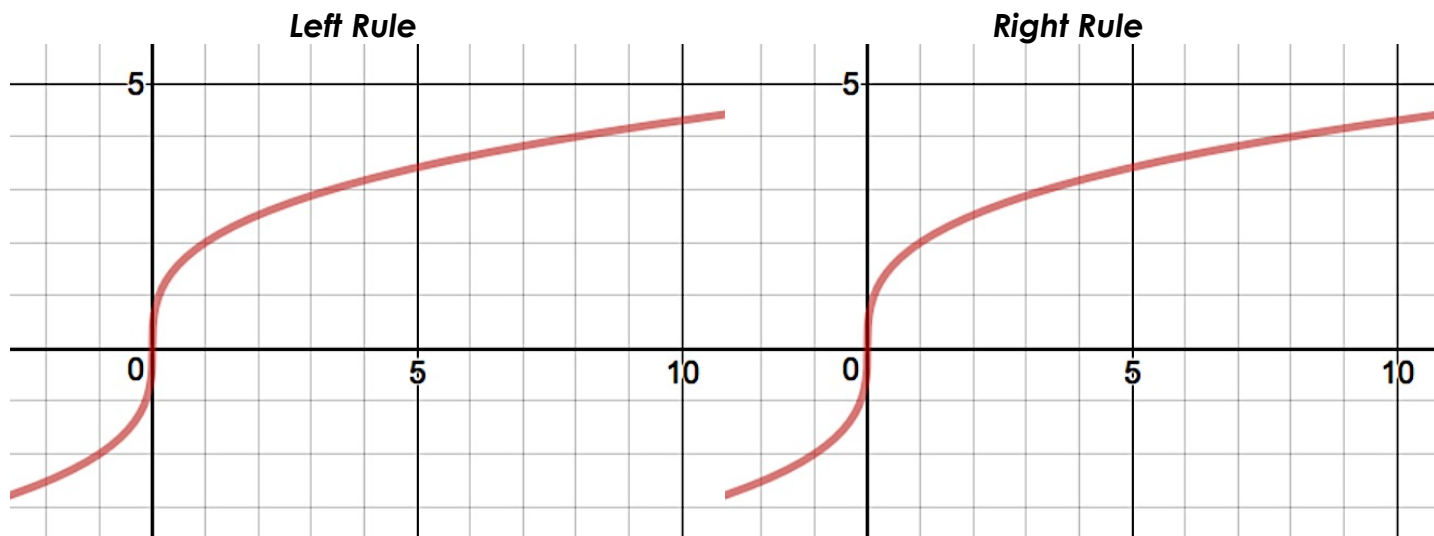


Calculus Lesson 423: Approximating the Area under a curve

Riemann Sums: Midpoint and Trapezoid Rules

Let $f(x) = 2x^{1/3}$

We will find the area under the curve of $f(x)$ from $x = 0$ to $x = 8$ using 4 subintervals by the Right Rule, Left Rule, Midpoint Rule, and Trapezoid Rule.



Rect.	Left	Right	Width	Height	Area
1					
2					
3					
4					

Approximate total area =

Over or Under Estimate?

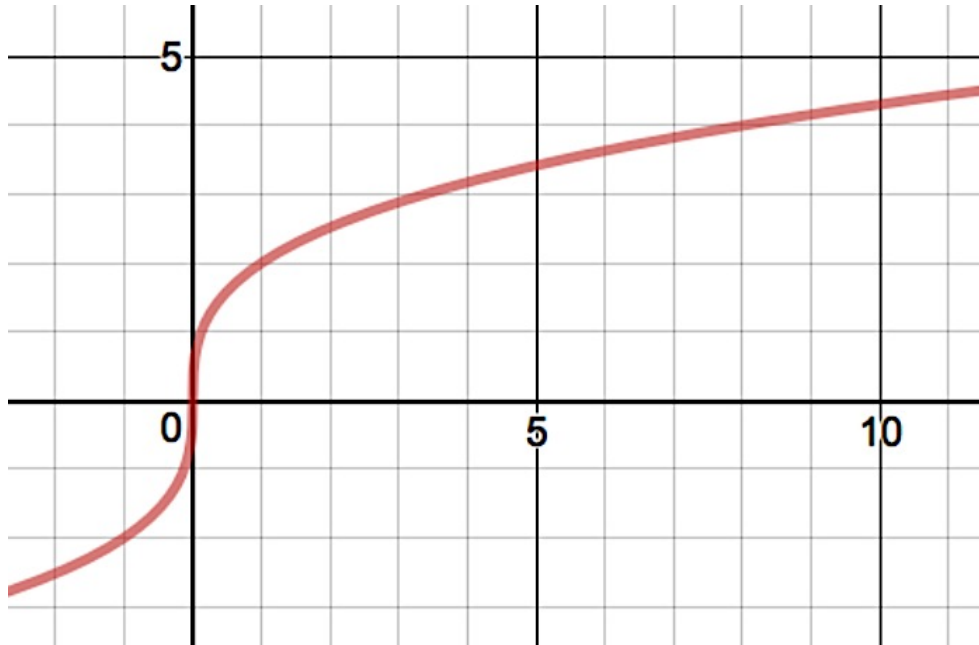
Rect.	Left	Right	Width	Height	Area
1					
2					
3					
4					

Approximate total area =

Over or Under Estimate?

Trapezoid Rule

$$\text{Area} = \frac{1}{2} (\text{Height at the Left} + \text{Height at the Right}) \cdot \text{Width}$$



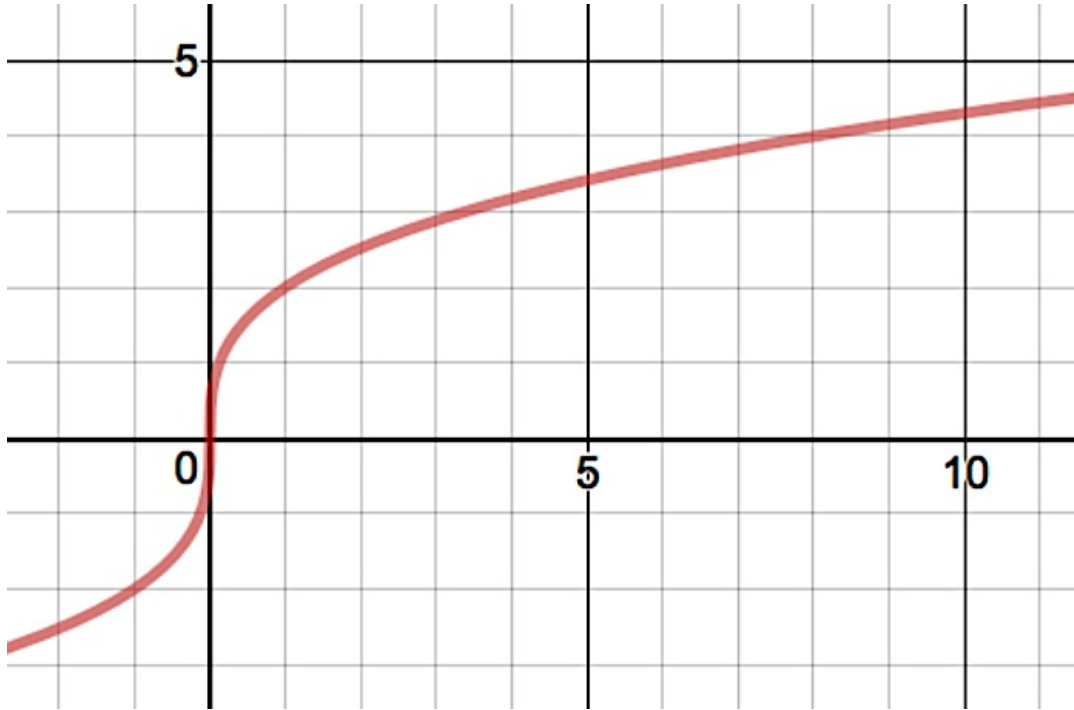
Trap	Left	Right	Width	Height at the Left	Height at the Right	Area
1						
2						
3						
4						

Approximate total area =

Over or Under Estimate?

Now Average your answers for the Left Rule and Right Rule from the previous page. What do you notice?

Midpoint Rule



Rect.	Left	Right	Width	Midpoint	Height at the Midpoint	Area
1						
2						
3						
4						

Approximate total area =

Over or Under Estimate?

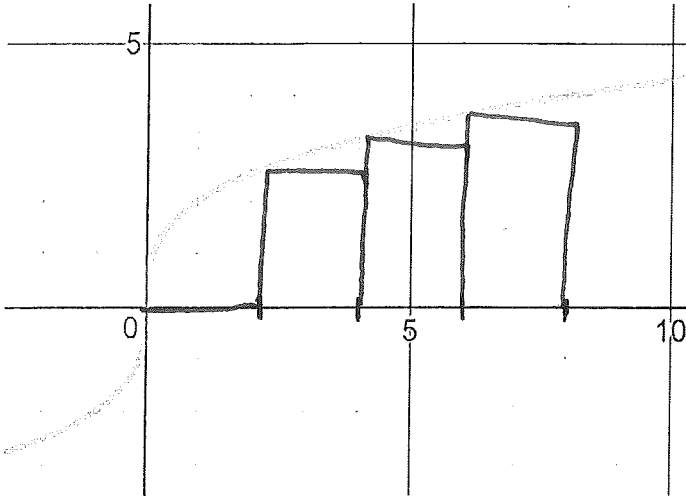
Calculus Lesson 423: Approximating the Area under a curve

Riemann Sums: Midpoint and Trapezoid Rules

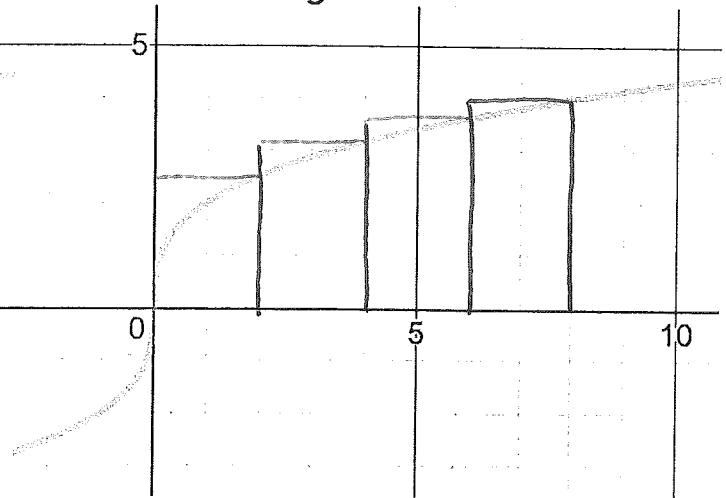
Let $f(x) = 2x^{1/3}$

We will find the area under the curve of $f(x)$ from $x = 0$ to $x = 8$ using 4 subintervals by the Right Rule, Left Rule, Midpoint Rule, and Trapezoid Rule.

Left Rule



Right Rule



Rect.	Left	Right	Width	Height	Area
1	0	2	2	0	0
2	2	4	2	2.5	5
3	4	6	2	3.2	6.4
4	6	8	2	3.6	7.2

Approximate total area = 18.7

Over or Under Estimate?

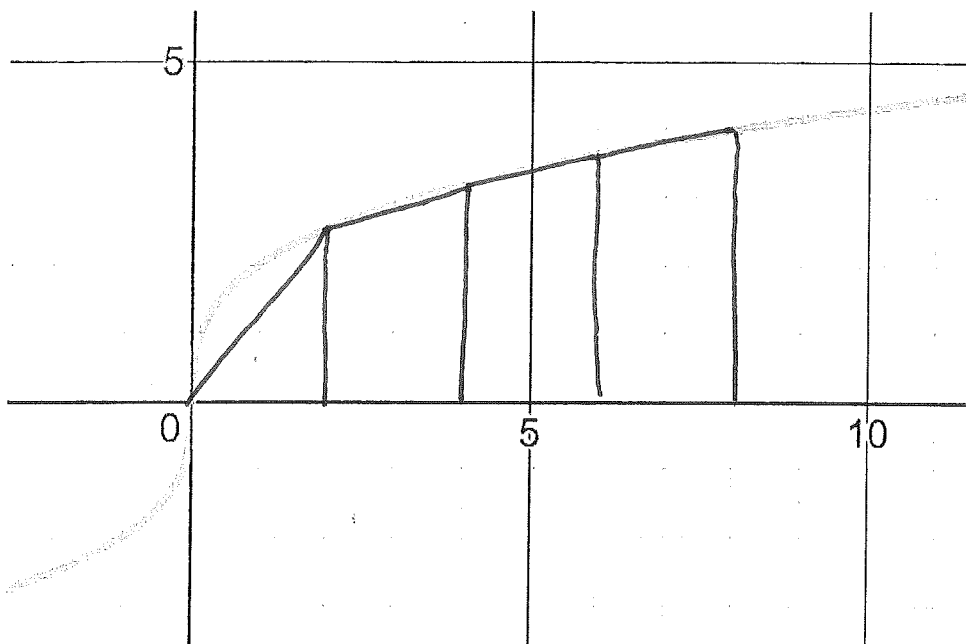
Rect.	Left	Right	Width	Height	Area
1	0	2	2	2.5	5
2	2	4	2	3.2	6.4
3	4	6	2	3.6	7.2
4	6	8	2	4	8

Approximate total area = 26.7

Over or Under Estimate?

Trapezoid Rule

$$\text{Area} = \frac{1}{2} (\text{Height at the Left} + \text{Height at the Right}) \cdot \text{Width}$$



Trap	Left	Right	Width	Height at the Left	Height at the Right	Area
1	0	2	2	0	2.5	
2	2	4	2	2.5	3.2	
3	4	6	2	3.2	3.6	
4	6	8	2	3.6	4	

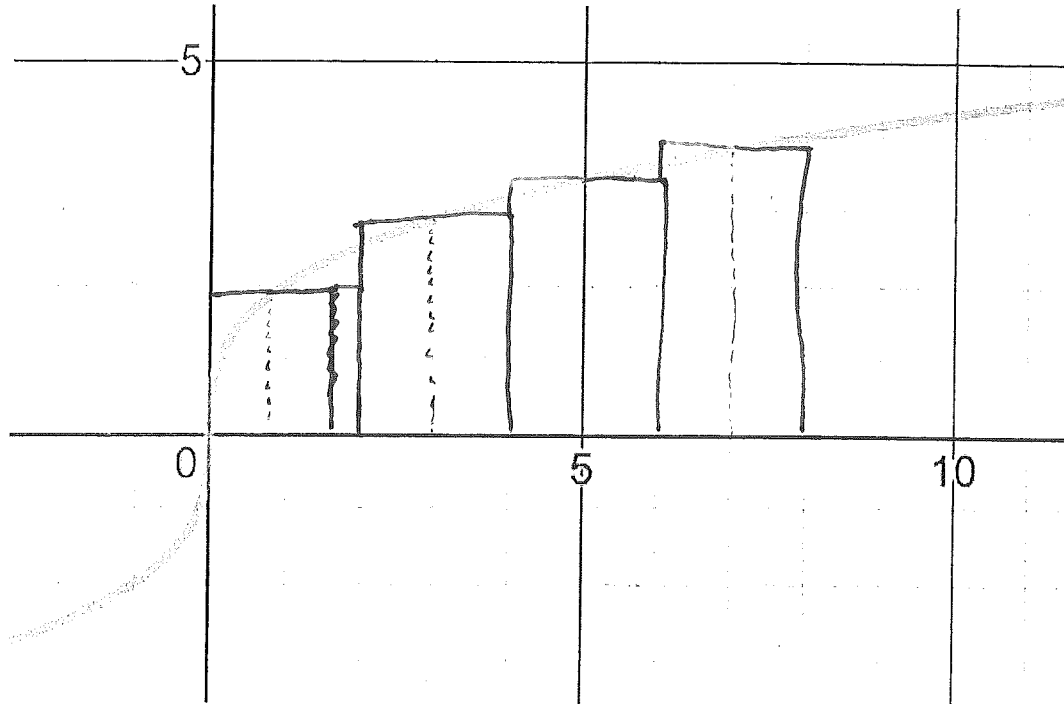
Approximate total area = 22.66

Over or Under Estimate?

Now Average your answers for the Left Rule and Right Rule from the previous page. What do you notice?

22.66

Midpoint Rule



Rect.	Left	Right	Width	Midpoint	Height at the Midpoint	Area
1	0	2	2	1	2	
2	2	4	2	3	2.88	
3	4	6	2	5	3.42	
4	6	8	2	7	3.83	

Approximate total area = 24.26

Over or Under Estimate?