

Calculus Lesson 422: Approximating the Area under a curve

Riemann Sums: Left and Right Rules

Part 1 - Consider calculating the area under the function $f(x) = -x^2 + 4$ from $x = 0$ to $x = 2$.

Shade this region.

The actual area is 5.333

Now use the **LEFT RULE**

Part 2 – Construct two rectangles of width 1 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1					
2					

Approximate total area =

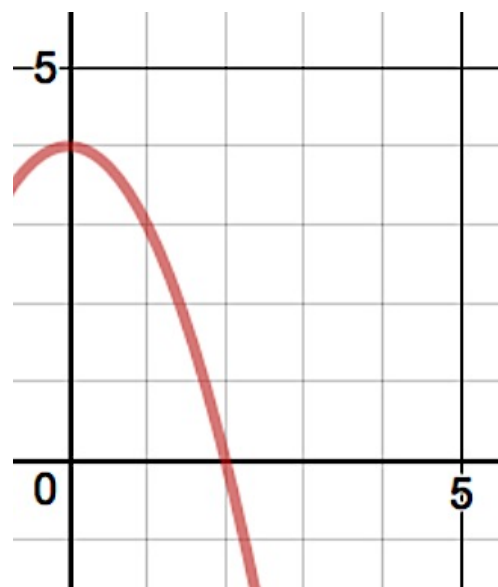
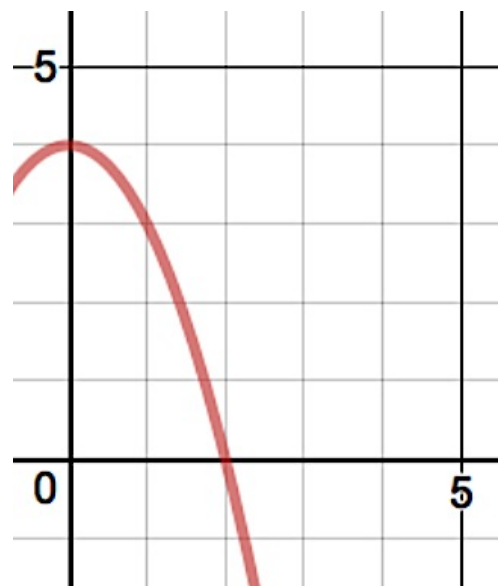
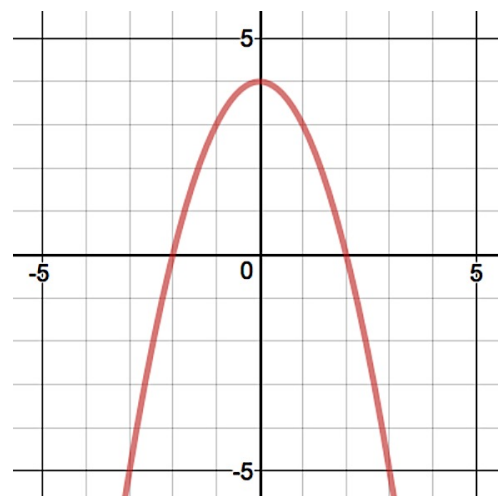
Over or Under Estimate?

Part 3 – Construct four rectangles of width 0.5 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1					
2					
3					
4					

Approximate total area =

Over or Under Estimate?



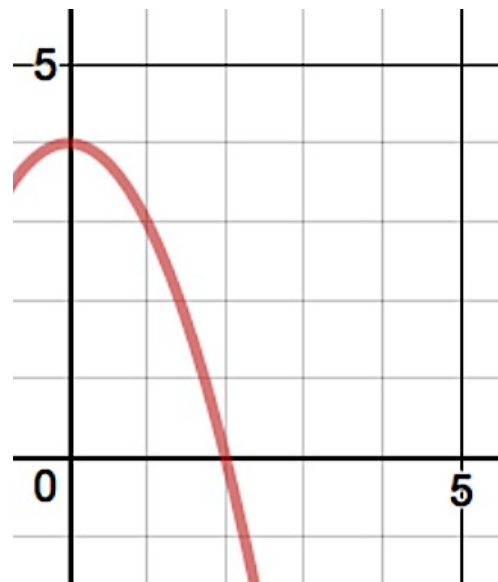
Now use the **RIGHT RULE**

Part 4 – Construct two rectangles of width 1 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1					
2					

Approximate total area =

Over or Under Estimate?

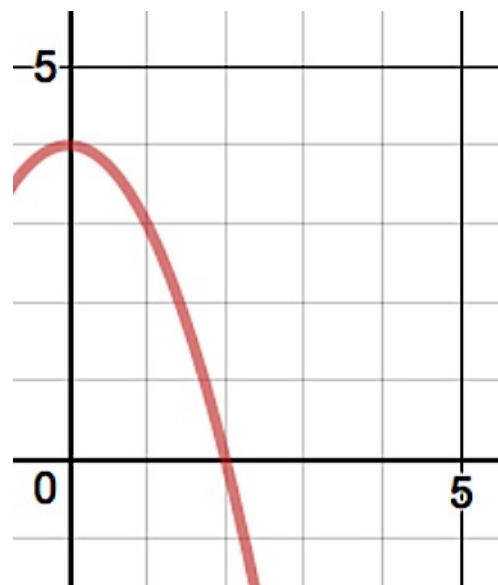


Part 5 – Construct four rectangles of width 0.5 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1					
2					
3					
4					

Approximate total area =

Over or Under Estimate?



Part 6 – Compare the approximation using 2 and 4 rectangles to the actual area. Which is more accurate?

To make the approximation more accurate, _____ the number of rectangles.

To make the approximation perfectly accurate, use _____ rectangles.

Part 7 – Consider $g(x) = \sqrt{x}$

If 4 rectangles are used to approximate the area under the curve of $g(x)$ from $x = 1$ to $x = 6$ then what is the width of each rectangle?

If 10 rectangles are used to approximate the area under the curve of $g(x)$ from $x = 1$ to $x = 6$ then what is the width of each rectangle?

If n rectangles are used to approximate the area under the curve of $g(x)$ from $x = L$ to $x = R$ then what is the width of each rectangle?

Part 8 - Now construct eight rectangles to approximate the area from $x = 0$ to $x = 2$ under the curve . The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Left Height	Area using Left Height	Right Height	Area using Right Height
1							
2							
3							
4							
5							
6							
7							
8							

Approximate total areas

Left Rule =

Over or Under Estimate?

Right Rule =

Over or Under Estimate?

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Shade this region.

The actual area is 5.333

Now use the **LEFT RULE**

Part 2 - Construct two rectangles of width 1 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y-value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1	0	1	1	4	4
2	1	2	1	3	3

Approximate total area = 7

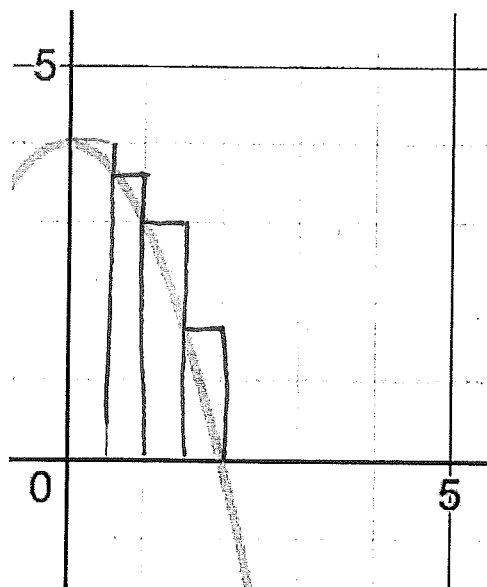
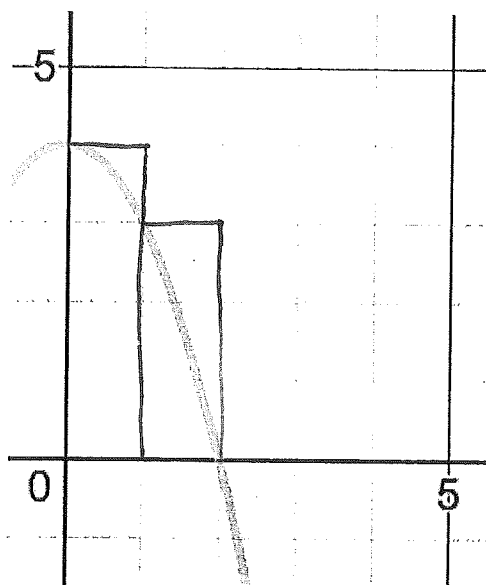
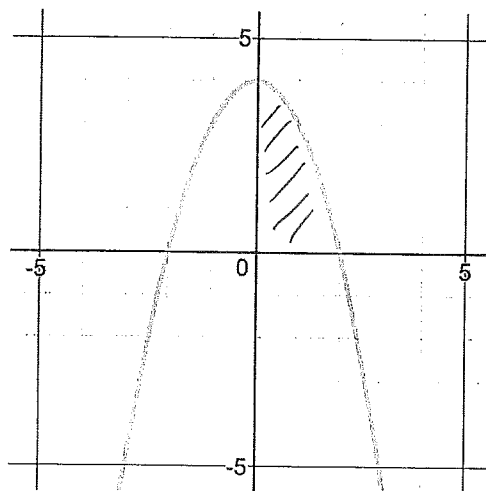
Over or Under Estimate?

Part 3 - Construct four rectangles of width 0.5 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y-value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1	0	$\frac{1}{2}$	$\frac{1}{2}$	4	2
2	$\frac{1}{2}$	1	$\frac{1}{2}$	3.75	1.875
3	1	$\frac{3}{2}$	$\frac{1}{2}$	3	1.5
4	$\frac{3}{2}$	2	$\frac{1}{2}$	1.75	.875

Approximate total area = 6.25

Over or Under Estimate?



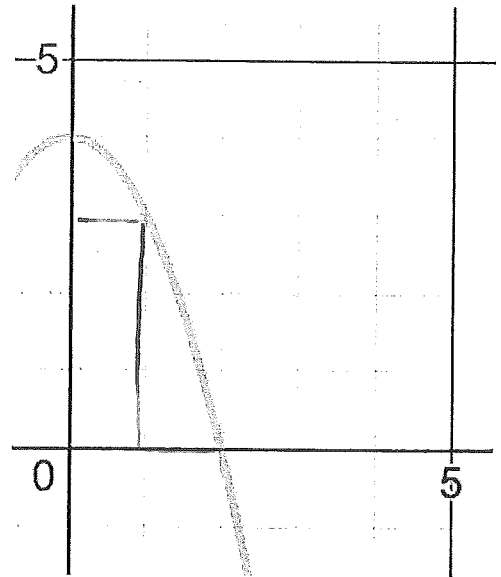
Now use the **RIGHT RULE**

Part 4 – Construct two rectangles of width 1 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1	0	1	1	3	3
2	1	2	1	0	0

Approximate total area = 3

Over or Under Estimate?

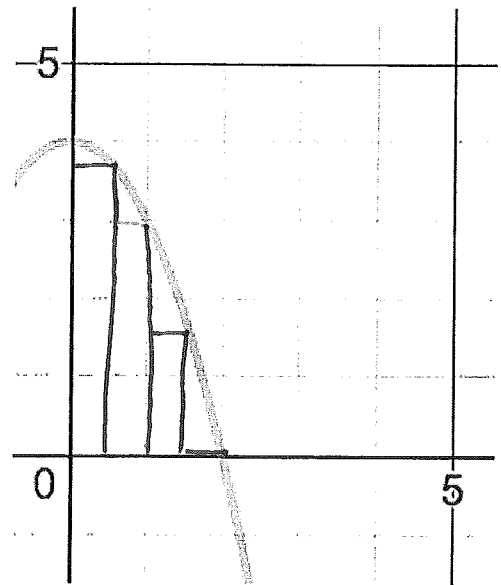


Part 5 – Construct four rectangles of width 0.5 to approximate the area from $x = 0$ to $x = 2$. The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Height	Area
1	0	$\frac{1}{2}$	$\frac{1}{2}$	3.75	1.875
2	$\frac{1}{2}$	1	$\frac{1}{2}$	3	1.5
3	1	$\frac{3}{2}$	$\frac{1}{2}$	1.75	.875
4	$\frac{3}{2}$	2	$\frac{1}{2}$	0	0

Approximate total area = 4.25

Over or Under Estimate?



Part 6 – Compare the approximation using 2 and 4 rectangles to the actual area. Which is more accurate? 4

To make the approximation more accurate, increase the number of rectangles.

To make the approximation perfectly accurate, use ∞ rectangles.

Part 7 - Consider $g(x) = \sqrt{x}$

If 4 rectangles are used to approximate the area under the curve of $g(x)$ from $x = 1$ to $x = 6$ then what is the width of each rectangle?

$$\frac{6-1}{4} = 1.25$$

If 10 rectangles are used to approximate the area under the curve of $g(x)$ from $x = 1$ to $x = 6$ then what is the width of each rectangle?

$$\frac{6-1}{10} = .5$$

If n rectangles are used to approximate the area under the curve of $g(x)$ from $x = L$ to $x = R$ then what is the width of each rectangle?

$$\text{width} = \frac{R-L}{n}$$

Part 8 - Now construct eight rectangles to approximate the area from $x = 0$ to $x = 2$ under the curve. The height of each rectangle will be found using the y -value at the left side of each rectangle.

Rect.	Left	Right	Width	Left Height	Area using Left Height	Right Height	Area using Right Height
1			.25				
2			↓				
3							
4							
5							
6							
7							
8							

Approximate total areas

Left Rule = 1.685

Over or Under Estimate?

Right Rule = 2.038

Over or Under Estimate?

Actual = 1.886