# Calculus Lesson 421: Sigma Notation

## Sigma Notation

- Sigma notation is a succinct way to write a long sum of numbers that follow a pattern.
- Basic Form:

$$\sum_{i=start}^{end} equation\ using\ i$$

**Find the following sums**

$$\sum_{i=1}^{6} 3i$$

$$\sum_{i=0}^{5} (i+1)$$

$$\sum_{j=3}^{7} j^2$$

$$\sum_{i=1}^{n} \frac{1}{n}(i^2+1)$$

**Write the following sums using sigma notation with start *i* = 1:**

A. -2 + -1 + 0 + 1

B. 1 + 4 + 9 + 16 + 25 + 36

C. 7 + 10 + 13 + 16 + 19 + 22 + 25

D. 8 + 6 + 4 + 2 + 0 + -2 + -4

E. -1 + 0 + 1 + 8 + 27 + 64

F. 4 – 9 + 14 – 19 + 24 – 29

**Sequences on the Calculator:**

- Use the sequence command to show the list of numbers
- The seq( command is found by pressing 2nd LIST and it is the 5th choice in the OPS menu.
- Example: State the first 5 terms for the sequence described by $a_i = i^2 - i$

seq($x^2$ – x, x, 1, 5, 1)

**Sums on the Calculator:**

- Use the sum command to find the answer to the sum.
- The sum( command is found by pressing 2nd LIST and it is the 5th choice in the MATH menu.
- Example: State the sum of the first 6 terms of the sequence described by 6x – 2.

sum(seq(6x – 2, x, 1, 6, 1))

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- Sigma notation is a succinct way to write a long sum of numbers that follow a pattern.
- Basic Form:

$$\sum_{i=start}^{end} equation\ using\ i$$

### Find the following sums

$$\sum_{i=1}^{6} 3i = 3(1)+3(2)+3(3)+3(4)+3(5)+3(6) = 63$$

$$\sum_{i=0}^{5} (i+1) = 1+2+3+4+5+6 = 21$$

$$\sum_{j=3}^{7} j^2 = 9+16+25+36+49 = 135$$

$$\sum_{i=1}^{n} \frac{1}{n}(i^2+1) = \frac{1}{n}(1^2+1) + \frac{1}{n}(2^2+1) + \cdots + \frac{1}{n}(n^2+1)$$

### Write the following sums using sigma notation with start *i* = 1:

A. -2 + -1 + 0 + 1

linear

$$\sum_{i=1}^{4} (i-3)$$

B. 1 + 4 + 9 + 16 + 25 + 36

quad

$$\sum_{i=1}^{6} i^2$$

C. 7 + 10 + 13 + 16 + 19 + 22 + 25

linear

$$\sum_{i=1}^{7} (3i+4)$$

D. 8 + 6 + 4 + 2 + 0 + -2 + -4

linear

$$\sum_{i=1}^{7} (-2i+10)$$

E. -1 + 0 + 1 + 8 + 27 + 64

cubic

$$\sum_{i=1}^{6} (i-2)^3$$

F. 4 – 9 + 14 – 19 + 24 – 29

numbers linear

signs swap

$$\sum_{i=1}^{6} (-1)^{i+1} \cdot (5i-1)$$

**In Exercises 1–6, find the sum. Use the summation capabilities of a graphing utility to verify your result.**

1. $\sum_{i=1}^{5}(2i+1)$

2. $\sum_{k=3}^{6}k(k-2)$

3. $\sum_{k=0}^{4}\frac{1}{k^2+1}$

4. $\sum_{j=3}^{5}\frac{1}{j}$

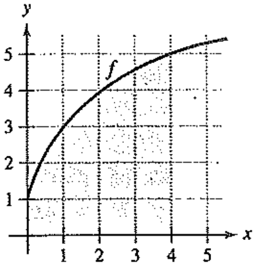
**In Exercises 7–14, use sigma notation to write the sum.**

7. $\frac{1}{3(1)}+\frac{1}{3(2)}+\frac{1}{3(3)}+\cdots+\frac{1}{3(9)}$

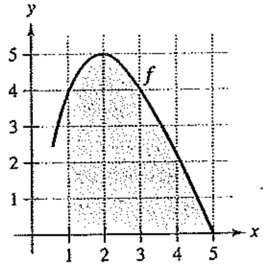
8. $\frac{5}{1+1}+\frac{5}{1+2}+\frac{5}{1+3}+\cdots+\frac{5}{1+15}$

**In Exercises 23–26, bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.**

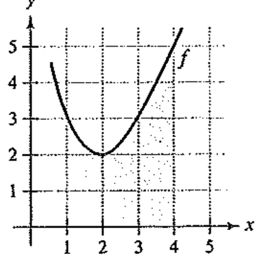
23. Left



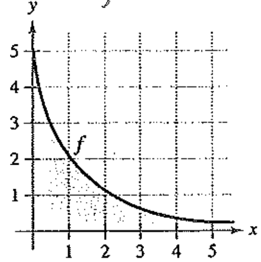

24. Left




25. Right

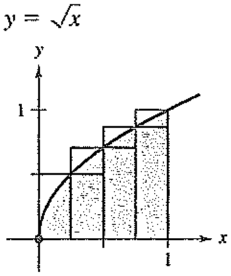



26. Right

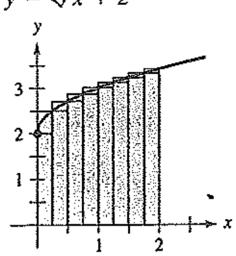



**In Exercises 27–30, use upper and lower sums to approximate the area of the region using the indicated number of subintervals (of equal width).**

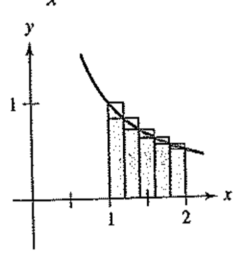
27. $y=\sqrt{x}$



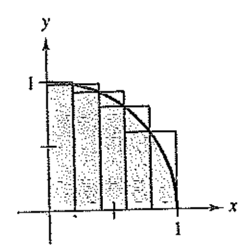

28. $y=\sqrt{x}+2$




29. $y=\frac{1}{x}$




30. $y=\sqrt{1-x^2}$



Calculus Worksheet 4-2A Name Key Pd

In Exercises 1–6, find the sum. Use the summation capabilities of a graphing utility to verify your result.

1. $\sum_{i=1}^{5}(2i+1)$ 2. $\sum_{k=3}^{6}k(k-2)$

3. $\sum_{k=0}^{4}\frac{1}{k^2+1}$ 4. $\sum_{j=3}^{5}\frac{1}{j}$

① $(2\cdot1+1)+(2\cdot2+1)+(2\cdot3+1)+(2\cdot4+1)+(2\cdot5+1)$
$3+5+7+9+11=35$

② $(3(3-2))+(4(4-2))+(5(5-2))+(6(6-2))$
$3+8+15+24=50$

③ $\frac{1}{0^2+1}+\frac{1}{1^2+1}+\frac{1}{2^2+1}+\frac{1}{3^2+1}+\frac{1}{4^2+1}$
$1+\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}=\frac{158}{85}$

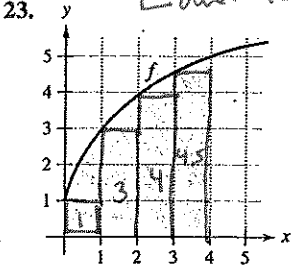
In Exercises 7–14, use sigma notation to write the sum.

7. $\frac{1}{3(1)}+\frac{1}{3(2)}+\frac{1}{3(3)}+\cdots+\frac{1}{3(9)}$ $\sum_{i=1}^{9}\frac{1}{3i}$

8. $\frac{5}{1+1}+\frac{5}{1+2}+\frac{5}{1+3}+\cdots+\frac{5}{1+15}$ $\sum_{i=1}^{15}\frac{5}{1+i}$

In Exercises 23–26, bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.

23. Lower (Left)

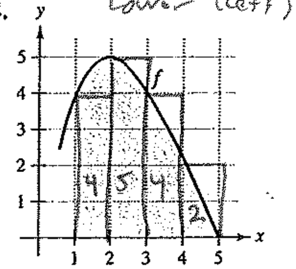


1 3 4 4.5 5

~~Upper 3+4+4.5+5~~

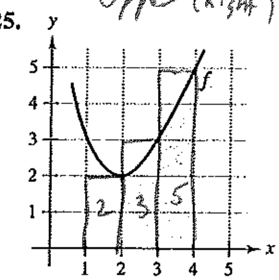
Lower: $1+3+4+4.5=12.5$

24. Lower (Left)



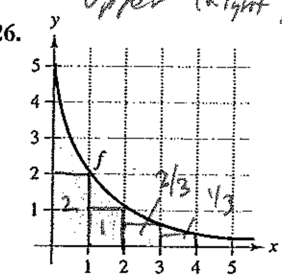
$4+5+4+2$
15

25. Upper (Right)



$2+3+5$
10

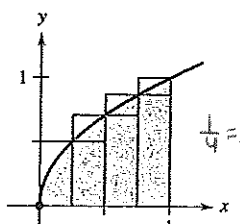
26. Upper (Right)



$2+1+\frac{2}{3}+\frac{1}{3}$
4

In Exercises 27–30, use upper and lower sums to approximate the area of the region using the indicated number of subintervals (of equal width).

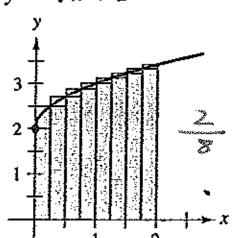
27. $y=\sqrt{x}$ L.



$\frac{1}{4}=.25$

Left $.25\sum_{i=0}^{.75}\sqrt{i}=\sqrt{0}+\sqrt{.25}+\sqrt{.5}+\sqrt{.75}$
$.25[0+.5+.7071+.866]$
.518

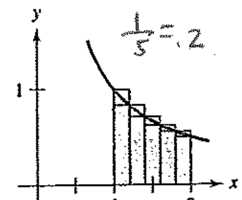
28. $y=\sqrt{x}+2$ L.



$\frac{2}{8}=.25$

$.25\sum_{i=0}^{1.75}\sqrt{i}+2$
$.25[0+2+\sqrt{.25}+2+\sqrt{.5}+2+\cdots+\sqrt{1.75}+2]$
5.684

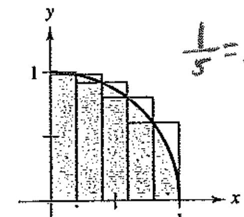
29. $y=\frac{1}{x}$ R.



$\frac{1}{5}=.2$

$\sum_{i=1.2}^{2}\frac{.2}{i}=\frac{.2}{1.2}+\frac{.2}{1.4}+\frac{.2}{1.6}+\frac{.2}{1.8}+\frac{.2}{2}$
.645

30. $y=\sqrt{1-x^2}$ R.



$\frac{1}{5}=.2$

$\sum_{i=.2}^{1}\sqrt{1-i^2}(.2)$
$.2[\sqrt{1-.2^2}+\sqrt{1-.4^2}+\sqrt{1-.6^2}+\sqrt{1-.8^2}+\sqrt{1-1^2}]=$
.659