

Calculus Lesson 391 Differential Equations

Differentials

dx is called the differential of x

dy is called the differential of y

Let $y = f(x)$ represent a function that is differentiable in an open interval containing x .

$$dy = f'(x) dx$$

Differentials are used for approximations and for multiple applications in engineering.

Example – Find the differential dy for the following

11. $y = 3x^2 - 4$

12. $y = 3x^{2/3}$

Method 1, use the Formula:

$$f(x + \Delta x) = f(x) + f'(x)dx$$

Δx and dx are both the distance from the x used to create the tangent line

Method 2: find the equation of the tangent line and use it to calculate the approximation.

Find the tangent line of $y = 1 + \sin(x)$ at the point $(0, 1)$ and use it to approximate the y -value at $x = .01$.

Find the tangent line at the given point. Then use the tangent line to approximate the y -value when $\Delta x = .01$

$$f(x) = x(8-x) \text{ at } (3, 15)$$

Find the differential dy for the following

13. $y = \frac{x+1}{2x-1}$

15. $y = x\sqrt{1-x^2}$

17. $y = 2x - \cot^2 x$

19. $y = \frac{1}{3} \cos\left(\frac{6\pi x - 1}{2}\right)$

Find the tangent line at the given point. Then use the tangent line to approximate the y -value when $\Delta x = .01$

<u>Function</u>	<u>Point</u>
1. $f(x) = x^2$	$(2, 4)$

4. $f(x) = \sqrt{x}$ $(2, \sqrt{2})$

5. $f(x) = \sin x$ $(2, \sin 2)$

In Exercises 7–10, use the information to evaluate and compare Δy and dy .

7. $y = \frac{1}{2}x^3$ $x = 2$ $\Delta x = dx = 0.1$

8. $y = 1 - 2x^2$ $x = 0$ $\Delta x = dx = -0.1$

Match the functions in Problems 21–24 with one of the derivatives in Figure 2.24.

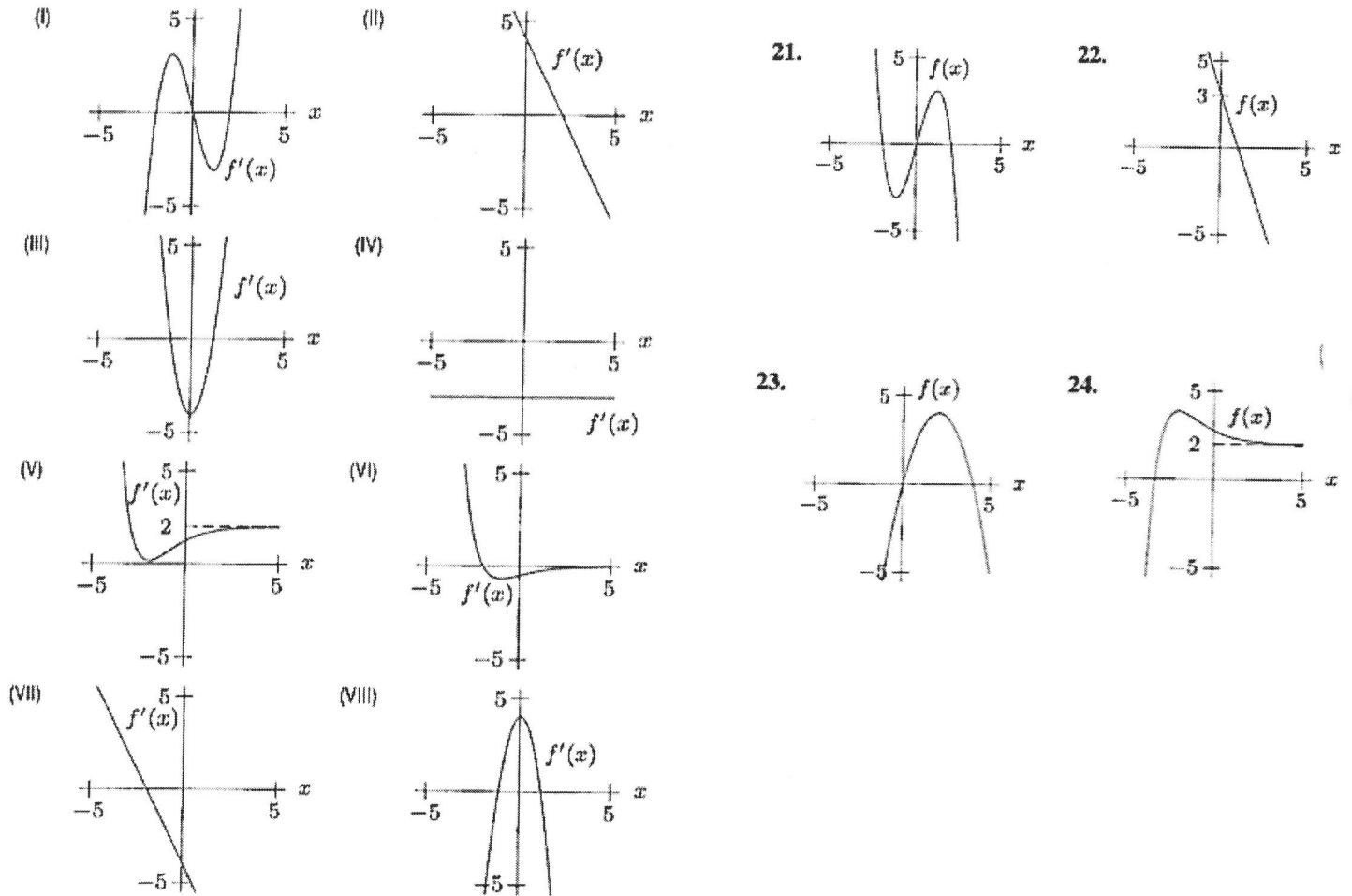
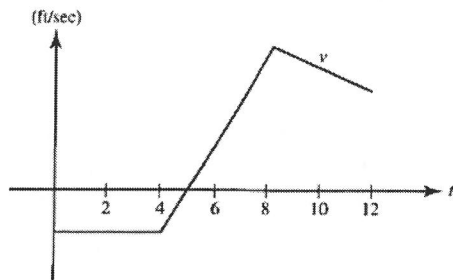


Figure 2.24

The graph for Questions 82 and 83 shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 12$.



82. For what t does this object attain its maximum acceleration?
- (A) $0 < t < 4$ (B) $4 < t < 8$ (C) $t = 5$ (D) $t = 8$ (E) $t = 12$
83. The object reverses direction at $t =$
- (A) 4 only (B) 5 only (C) 8 only
 (D) 5 and 8 (E) none of these