

Calculus Lesson 15.1 Limits with Infinite Outputs

Method 2 – function analysis

How can you tell from a function that the y-value approaches infinity (vertical asymptote)?

Consider: $f(x) = x / (x^2 - x)$

Parent Function:

Holes:

Vertical Asymptotes:

Picture:

Consider #16: $h(s) = (2s - 3) / (s^2 - 25)$

Parent Function:

Holes:

Vertical Asymptotes:

When is the top zero:

Table to consider Positives and Negatives:

Picture:

In Exercises 9–28, find the vertical asymptotes (if any) of the function.

17. $f(x) = \tan 2x$

18. $f(x) = \sec \pi x$

19. $T(t) = 1 - \frac{4}{t^2}$

20. $g(x) = \frac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$

21. $f(x) = \frac{x}{x^2 + x - 2}$

22. $f(x) = \frac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$

23. $g(x) = \frac{x^3 + 1}{x + 1}$

24. $h(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2}$

25. $f(x) = \frac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

26. $h(t) = \frac{t^2 - 2t}{t^4 - 16}$

27. $s(t) = \frac{t}{\sin t}$

28. $g(\theta) = \frac{\tan \theta}{\theta}$

Use Calculator for 29–32 and 49–52


In Exercises 29–32, determine whether the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

29. $f(x) = \frac{x^2 - 1}{x + 1}$

30. $f(x) = \frac{x^2 - 6x - 7}{x + 1}$

31. $f(x) = \frac{x^2 + 1}{x + 1}$

32. $f(x) = \frac{\sin(x + 1)}{x + 1}$

 In Exercises 49–52, use a graphing utility to graph the function and determine the one-sided limit.

49. $f(x) = \frac{x^2 + x + 1}{x^3 - 1}$

50. $f(x) = \frac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x \rightarrow 1^+} f(x)$

$\lim_{x \rightarrow 1^-} f(x)$

51. $f(x) = \frac{1}{x^2 - 25}$

52. $f(x) = \sec \frac{\pi x}{6}$

$\lim_{x \rightarrow 5^-} f(x)$

$\lim_{x \rightarrow 3^+} f(x)$

In Exercises 33–48, find the limit.

33. $\lim_{x \rightarrow 2^+} \frac{x - 3}{x - 2}$

35. $\lim_{x \rightarrow 3^+} \frac{x^2}{x^2 - 9}$

37. $\lim_{x \rightarrow -3^-} \frac{x^2 + 2x - 3}{x^2 + x - 6}$

39. $\lim_{x \rightarrow 1} \frac{x^2 - x}{(x^2 + 1)(x - 1)}$

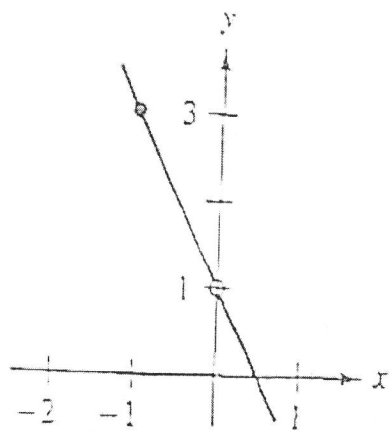
41. $\lim_{x \rightarrow 0^-} \left(1 + \frac{1}{x}\right)$

43. $\lim_{x \rightarrow 0^+} \frac{2}{\sin x}$

45. $\lim_{x \rightarrow \pi} \frac{\sqrt{x}}{\csc x}$

47. $\lim_{x \rightarrow 1/2} x \sec \pi x$

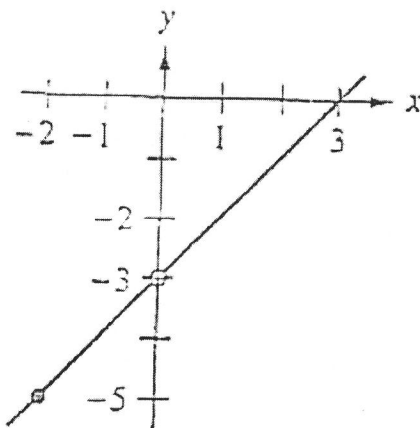
$$41. g(x) = \frac{-2x^2 + x}{x}$$



(a) $\lim_{x \rightarrow 0} g(x)$

(b) $\lim_{x \rightarrow -1} g(x)$

$$42. h(x) = \frac{x^2 - 3x}{x}$$



(a) $\lim_{x \rightarrow -2} h(x)$

(b) $\lim_{x \rightarrow 0} h(x)$