Calculus Lesson 151 Limits with Infinite Outputs

Key Questions:

How can you tell from a table that the y-value approaches infinity (vertical asymptote)?

Consider: $f(x) = 3/(x+1)$ as $x \to -1$

x	-1.1	-1.01	-1.001	Left hand limit
f(x)				

x	-0.9	-0.99	-0.999	Right hand limit
f(x)				

Overall limit:

Picture:

Method 1: Get out of jail free – plug in values infinitely close to the limit input.

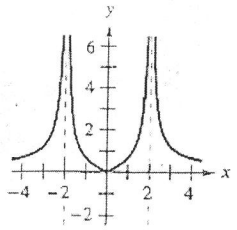
Example: $f(x) = 5x / (x - 2)$ Find the limit as $x \to 2$

Your Turn: $f(x) = x^3 / (x + 1)^3$ Find the limit as $x \to -1$

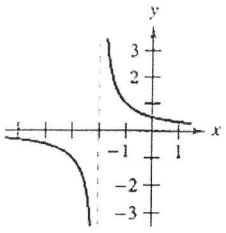
Calculus Worksheet 151 Name_____ Pd ____

In Exercises 1–4, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

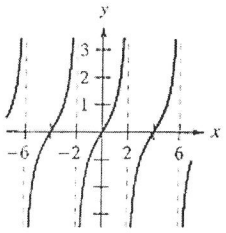
1. $f(x) = 2\left|\dfrac{x}{x^2 - 4}\right|$



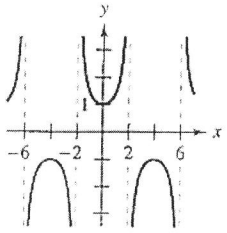
2. $f(x) = \dfrac{1}{x + 2}$



3. $f(x) = \tan\dfrac{\pi x}{4}$



4. $f(x) = \sec\dfrac{\pi x}{4}$



	Left Limit	Right Limit	Final Limit
#1			
#2			
#3			
#4			

Use Calculator for 5-8

Numerical and Graphical Analysis In Exercises 5–8, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the table. Use a graphing utility to graph the function and confirm your answer.

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

5. $f(x) = \dfrac{1}{x^2 - 9}$

6. $f(x) = \dfrac{x}{x^2 - 9}$

7. $f(x) = \dfrac{x^2}{x^2 - 9}$

8. $f(x) = \sec\dfrac{\pi x}{6}$

x→	-3.01	-3.001	Left Limit	-2.99	-2.999	Right Limit	Final Limit
#5							
#6							
#7							
#8							

In Exercises 9–28, find the vertical asymptotes (if any) of the function.

9. $f(x) = \dfrac{1}{x^2}$

10. $f(x) = \dfrac{4}{(x - 2)^3}$

11. $h(x) = \dfrac{x^2 - 2}{x^2 - x - 2}$

12. $g(x) = \dfrac{2 + x}{x^2(1 - x)}$

13. $f(x) = \dfrac{x^2}{x^2 - 4}$

14. $f(x) = \dfrac{-4x}{x^2 + 4}$

15. $g(t) = \dfrac{t - 1}{t^2 + 1}$

16. $h(s) = \dfrac{2s - 3}{s^2 - 25}$

4. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$

(A) 1 (B) $\frac{8}{-}$ (C) -1 (D) 0 (E) undefined

C

31. $\lim_{h \to 0} \dfrac{\tan\left(\frac{\pi}{6} + h\right) - \tan\left(\frac{\pi}{6}\right)}{h} =$

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\frac{3}{4}$

20. The function f is given by $f(x) = \begin{cases} \ln 2x, & 0 < x < 2 \\ 2 \ln x, & x \geq 2 \end{cases}$

The limit $\lim_{x \to 2} f(x)$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $2 \ln 2$

(E) nonexistent

Hint: log(4^3) = 3log(4)