Lesson 142

Section 1.4 Intermediate Value Theorem

Key Questions:

What does it mean to say that a function is continuous between 0 and 5?

Draw a graph of a continuous function on the interval 0 to 5 that contains the points (1, 3) and (4, -1) that does NOT have a zero.

Intermediate Value Theorem (IVT)

If f is continuous on the closed interval [a, b] and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in [a, b] such that $f(c) = k$.

Simpler:

Consider the interval [0, 5] you considered above.
The two points given were (1, 3) and (4, -1). This can also be written f(1) = 3 and f(4) = -1.
Since the y values are 3 and -1, then all the y values between 3 and -1 will be defined by some x value between the x values of 1 and 4.

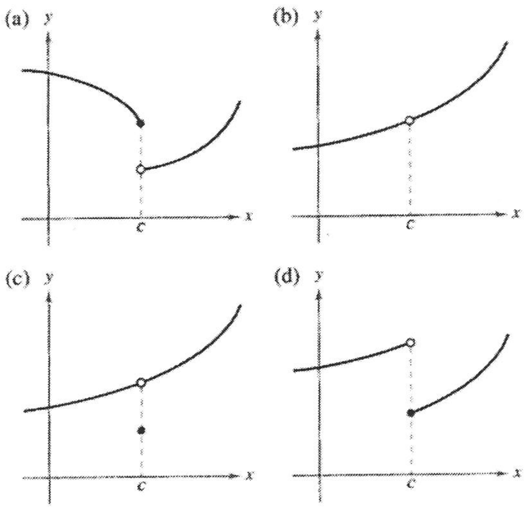
Simplest:

Since the point (1, 3) has a positive y-value and the point (4, -1) has a negative y-value there has to be a zero (hits the x-axis) between the x values of 1 and 4.

Picture:

Calculus **Worksheet ~~1.4B~~ 142** **Name**____________________________________ **Pd**_____

87. State how continuity is destroyed at $x = c$ for each of the following.



In Exercises 7–24, find the limit (if it exists). If it does not exist, explain why.

15. $\lim_{x \to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$

16. $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

17. $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

18. $\lim_{x \to 1^+} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$

In Exercises 33–54, find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

33. $f(x) = x^2 - 2x + 1$

35. $f(x) = 3x - \cos x$

37. $f(x) = \dfrac{x}{x^2 - x}$

39. $f(x) = \dfrac{x}{x^2 + 1}$

41. $f(x) = \dfrac{x+2}{x^2 - 3x - 10}$

43. $f(x) = \dfrac{|x+2|}{x+2}$

45. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

47. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

7. Find k so that $f(x)=\begin{cases}\dfrac{x^2-16}{x-4} & ;x\neq 4\\ k & ;x=4\end{cases}$ is continuous for all x.

(A) All real values of k make $f(x)$ continuous for all x.
(B) 0
(C) 16
(D) 8
(E) There is no real value of k that makes $f(x)$ continuous for all x.

15. If $f(x)=\dfrac{x^2+5x-24}{x^2+10x+16}$, then $\lim\limits_{x\to -8} f(x)$ is

(A) 0 (B) 1 (C) $-\frac{3}{2}$ (D) $\frac{11}{6}$ (E) Nonexistent

20. The function f is given by $f(x)=\begin{cases}\ln 2x, & 0<x<2\\ 2\ln x, & x\geq 2\end{cases}$.
The limit $\lim\limits_{x\to 2} f(x)$ is

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $2\ln 2$
(E) nonexistent