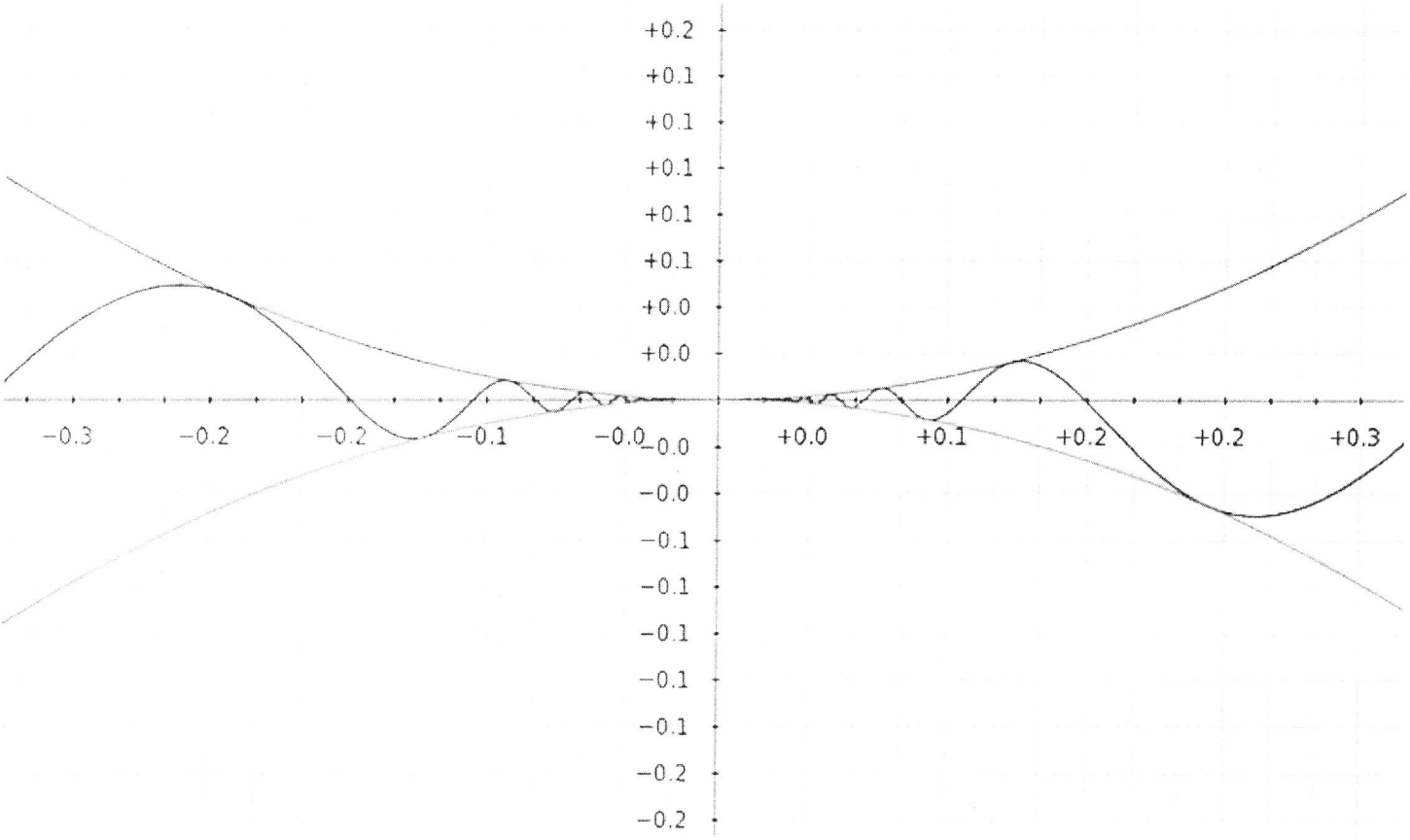
Squeeze Theorem:

If we know a function is between two other functions, then we can find the limits using the other functions. This is useful if the limit at an x-value is easy to compute on the other functions.

Example: in the picture below, the top function is $y = 0.1x^2$
the bottom function is $y = -0.1x^2$
the middle function is g(x)



if $-.1x^2 \le g(x) \le .1x^2$ for all x values then find the limit as $x \to 0$ on g(x)

bottom limit: limit as $x \to 0$ on $-.1x^2$ is 0

top limit: limit as $x \to 0$ on $.1x^2$ is 0

Middle limit: The y-value is between 0 and 0. Therefore the limit on g(x) must be 0.

Examples:

37. If $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for $x \geq 0$, find $\lim_{x \to 4} f(x)$.

38. If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x, evaluate $\lim_{x \to 1} g(x)$.

Your Turn:

41–46 Find the limit, if it exists. If the limit does not exist, explain why.

41. $\lim_{x \to 3} \left(2x + |x - 3|\right)$

42. $\lim_{x \to -6} \dfrac{2x + 12}{|x + 6|}$

HW: complete worksheet 134

In the following exercises, use the squeeze theorem to prove the limit.

223. $\lim_{x \to 0} x^2 \cos(2\pi x) = 0$

224. $\lim_{x \to 0} x^3 \sin\left(\frac{\pi}{x}\right) = 0$

225. Determine the domain such that the function $f(x) = \sqrt{x-2} + xe^x$ is continuous over its domain.

Review:

In the following exercises, sketch the graph of a function with the given properties.

76.

$\lim_{x \to 2} f(x) = 1$, $\lim_{x \to 4^-} f(x) = 3$, $\lim_{x \to 4^+} f(x) = 6$, $x = 4$ is not defined.

82. A track coach uses a camera with a fast shutter to estimate the position of a runner with respect to time. A table of the values of position of the athlete versus time is given here, where x is the position in meters of the runner and t is time in seconds. What is $\lim_{t \to 2} x(t)$? What does it mean physically?

t (sec)	x (m)
1.75	4.5
1.95	6.1
1.99	6.42
2.01	6.58
2.05	6.9
2.25	8.5

98. $\lim_{h \to 0} \dfrac{\dfrac{1}{a+h} - \dfrac{1}{a}}{h}$, where a is a real-valued constant

99. $\lim_{\theta \to \pi} \dfrac{\sin\theta}{\tan\theta}$

100. $\lim_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$

101. $\lim_{x \to 1/2} \dfrac{2x^2 + 3x - 2}{2x - 1}$

102. $\lim_{x \to -3} \dfrac{\sqrt{x+4} - 1}{x+3}$