Calculus Lesson 132 Analyzing Limits with Algebra

Concept Question: What if the value creates a zero in the denominator or a negative under an even root (This is the same as saying the value is not an element of the domain)?
You Should:

A Strategy for Finding Limits

1. Learn to recognize which limits can be evaluated by direct substitution. (These limits are listed in Theorems 1.1 through 1.6.)
2. If the limit of $f(x)$ as x approaches c *cannot* be evaluated by direct substitution, try to find a function g that agrees with f for all x other than $x = c$. [Choose g such that the limit of $g(x)$ *can* be evaluated by direct substitution.]
3. Apply Theorem 1.7 to conclude *analytically* that
$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = g(c).$$
4. Use a *graph* or *table* to reinforce your conclusion.

Examples

39. $f(x) = \dfrac{\sqrt{x+5} - 3}{x - 4}$

$\lim_{x \to 4} f(x)$

40. $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

$\lim_{x \to 3} f(x)$

41. $f(x) = \dfrac{x - 9}{\sqrt{x} - 3}$

$\lim_{x \to 9} f(x)$

42. $f(x) = \dfrac{x - 3}{x^2 - 9}$

$\lim_{x \to 3} f(x)$

Your Turn

45. $\lim_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

46. $\lim_{x \to -1} \dfrac{2x^2 - x - 3}{x + 1}$

47. $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

48. $\lim_{x \to -1} \dfrac{x^3 + 1}{x + 1}$

HW: complete worksheet 132

Calculus Worksheet 132 Name_____ Pd_____

In Exercises 49–62, find the limit

49. $\lim_{x\to 5} \frac{x-5}{x^2-25}$

51. $\lim_{x\to -3} \frac{x^2+x-6}{x^2-9}$

53. $\lim_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$

50. $\lim_{x\to 2} \frac{2-x}{x^2-4}$

52. $\lim_{x\to 4} \frac{x^2-5x+4}{x^2-2x-8}$

54. $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

In the following exercises, use direct substitution to show that each limit leads to the indeterminate form 0/0. Then, evaluate the limit.

93. $\lim_{x\to 4} \frac{x^2-16}{x-4}$

94. $\lim_{x\to 2} \frac{x-2}{x^2-2x}$

95. $\lim_{x\to 6} \frac{3x-18}{2x-12}$

96. $\lim_{h\to 0} \frac{(1+h)^2-1}{h}$

97. $\lim_{t\to 9} \frac{t-9}{\sqrt{t}-3}$

Review:

6. $\lim_{x \to -2} x^3$

8. $\lim_{x \to -3} (3x + 2)$

10. $\lim_{x \to 1} (-x^2 + 1)$

12. $\lim_{x \to 1} (3x^3 - 2x^2 + 4)$

14. $\lim_{x \to -3} \frac{2}{x + 2}$

16. $\lim_{x \to 3} \frac{2x - 3}{x + 5}$

18. $\lim_{x \to 3} \frac{\sqrt{x + 1}}{x - 4}$

20. $\lim_{x \to 4} \sqrt[3]{x + 4}$

22. $\lim_{x \to 0} (2x - 1)^3$

In Exercises 27–36, find the limit of the trigonometric function.

27. $\lim_{x \to \pi/2} \sin x$

28. $\lim_{x \to \pi} \tan x$

29. $\lim_{x \to 2} \cos \frac{\pi x}{3}$

30. $\lim_{x \to 1} \sin \frac{\pi x}{2}$

31. $\lim_{x \to 0} \sec 2x$

32. $\lim_{x \to \pi} \cos 3x$

33. $\lim_{x \to 5\pi/6} \sin x$

34. $\lim_{x \to 5\pi/3} \cos x$

3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

(A) 0
(B) 10
(C) –10
(D) 5
(E) The limit does not exist.

23. $\lim_{x \to 0} 4 \frac{\sin x \cos x - \sin x}{x^2} =$

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined